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MORE ON CONCURRENT INTERVAL-RATIO SCHEDULES: A REPLICATION AND REVIEW

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It has been suggested that the failure to maximize reinforcement on concurrent variable-interval, variable-ratio schedules may be misleading. Inasmuch as response costs are not directly measured, it is possible that subjects are optimally balancing the benefits of reinforcement against the costs of responding. To evaluate this hypothesis, pigeons were tested in a procedure in which interval and ratio schedules had equal response costs. On a concurrent variable time (VT), variable ratio-time (VRT) schedule, the VT schedule runs throughout the session and the VRT schedule is controlled by responses to a changeover key that switches from one schedule to the other. Reinforcement is presented independent of response. This schedule retains the essential features of concurrent VI VR, but eliminates differential response costs for the two alternatives. It therefore also eliminates at least one significant ambiguity about the reinforcement maximizing performance. Pigeons did not maximize rate of reinforcement on this procedure. Instead, their times spent on the alternative schedules matched the relative rates of reinforcement, even when schedule parameters were such that matching earned the lowest possible overall rate of reinforcement. It was further shown that the observed matching was not a procedural artifact arising from the constraints built into the schedule.

Key words: choice, maximization, optimization, matching, melioration, concurrent interval-ratio schedules, reinforcement feedback functions, time allocation, pigeons, pied wagtails, rats

In a previous paper (Herrnstein & Heyman, 1979) we described the results of a study in which pigeons chose between variable-interval (VI) and variable-ratio (VR) reinforcement schedules. The schedules were available concurrently, and the pigeons switched between them, responding for a while at each one. The choice ratios, defined as the allocation of responses or time between the schedules, closely approximated reinforcement ratios, as predicted by the matching law (Herrnstein, 1970).

This aspect of the results was conveniently summarized by Baum's (1974) log-ratio formulation of the matching law:

$$\log(B_1/B_2) = a \log(R_1/R_2) + \log b, \quad (1)$$

where B_1 was time or responses at the VI schedule, B_2 was time or responses at the VR schedule, R_1 and R_2 were the respective reinforcement rates, and a and b were estimated parameters (i.e., slope and antilog of the intercept). The major finding was that the estimated parameters, a and b, were quite different from those predicted by reinforcement maximization theory. For example, the slope and intercept for the time ratios that would have maximized reinforcement rate were 0.72 and -0.38, whereas the observed slope and intercept were 1.04 and 0.11, values that more nearly approximate the simpler, fixed-parameter matching law ($a = 1.0$ and $\log b = 0$, referred to below as "normative" matching). Moreover, maximizing of reinforcement rate would require that the pigeons spend most of their time responding on the VR schedule; instead, they spent most of their time responding on the VI schedule.

Defenders of maximization theory (Green, Rachlin, & Hanson, 1983; Rachlin, Battalio, Kagel, & Green, 1981) countered that food reinforcement rate was not the only influence on behavior, and that if such factors as response cost or leisure (defined as time not pecking) had been included, the pigeons would prove to have been maximizing reinforcement. For example, the pigeons responded approximately twice as fast while on the ratio schedule, so that they may have preferred the in-

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Table 1

Order of conditions.

	VT (s)	VRT (s)	Sessions
1	50	30	56
2	30	30	26
3	50	15	105
4	50	30	115
5	30	30	108*
6	30	30	72**
7	100	30	49**

* 1.5-s COD was removed.

** 1.5-s COD was reinstated and color of houselight correlated with schedule was switched.

terval schedule because it provided more "leisure." We tried to take this possibility into account by weighting the interval reinforcements more heavily than the ratio reinforcements, and then fitting Equation 1 to the transformed reinforcement rates (Equation 9, Herrnstein & Heyman, 1979). Although the magnitude of the weighting factor was based on the pigeons' revealed preference for interval schedules of reinforcement, the adjustment did little to improve the case for maximization. Green, Kagel, and Battalio (1982), however, found a transformation that they said supports maximization theory. They showed that performance on concurrent VI VR would be consistent with maximization if it was assumed that time spent not responding (leisure) was 3.4 times more rewarding than time spent eating. For the food-deprived pigeons used in these experiments, this is a highly questionable assumption (see Baum & Nevin, 1981, for a similar criticism).

DeCarlo (1985) tested the response-cost hypothesis directly. His procedure retained the essential features of concurrent VI VR, but eliminated discrete responding. He used a concurrent procedure pitting a variable-time schedule against a time-based analogue of a variable-ratio schedule (called concurrent VT VRT here). Response costs, if any, were necessarily the same for the two schedules. In his procedure, the distribution of time that maximizes overall rate of reinforcement therefore corresponds to that for food-reinforcement maximization. His subjects did not maximize food reinforcement rate, but matched just as they did with concurrent VI VR.

DeCarlo's experiment provides a critical test of global maximization, for matching and maximizing predict quite different outcomes, without the ambiguities of estimating response costs. For some schedule parameters on concurrent VT VRT, matching is the *least* likely outcome according to maximization theory (see Discussion). The experiment described in this report used essentially the same procedure as DeCarlo's, with special attention to those schedule parameters that most sharply separate the predictions of matching and global maximization.

METHOD

Subjects

Five male White Carneaux pigeons served as subjects. They were maintained at approximately 80% of their free-feeding weights, and they had been used in previous operant conditioning studies.

Apparatus

The experiment was conducted in a standard operant conditioning chamber. In the front wall of the chamber was a recessed opening that provided access to a grain hopper. Above the opening, 23 cm from the floor and centered between the left and right walls, was a circular plastic response key (Gerbrands) that could be illuminated from the back with white light. The response key was operated by a force of more than 0.15 N, and each effective response produced a brief relay click. Suspended from one corner of the chamber was a rectangular aluminum frame with a 12- by 11-cm translucent plastic cover. The frame held green and red Christmas tree bulbs that were used as houselights. Presentation of experimental events and data collection were handled by a PDP® 8e computer.

Procedure

At the start of each session both the red and green houselights were on and the response key, designated the changeover key, was illuminated. The first response at the changeover key started the session timer and turned off one or the other houselights but not both, with equal probability. Subsequent changeover-key responses switched the houselight color (red to green or green to red). The houselight color signaled which reinforcement contingency was in effect. For the first five conditions, the "ratio-schedule" timer ran only while the red light was on, whereas the "interval-schedule" timer ran while either the red or green light was on. (Colors were reversed for the sixth and seventh conditions, but this description will use color terms as initially appropriate.) When a ratio interval elapsed, which could happen only with the red light on, the reinforcer was delivered immediately. The interval timer provided reinforcement in two ways. If an interval elapsed while the green light was on, the reinforcer was delivered immediately (except for a changeover delay noted below). However, if an interval elapsed while the red light was on, the reinforcer was held until the subject responded at the changeover key and put the green light back on. Note, then, that while the red light was on, time accumulated for both the ratio and interval schedule requirements, but while the green light was on, it accumulated only for the interval schedule requirement. In addition, there was the stipulation that a reinforcer could not be delivered during the green periods until 1.5 s had elapsed since the last changeover-key response. (No COD was arranged on the red, i.e., ratio, alternative because no reinforcer could set up during visits to the other alternative.) The schedule, then, is a time-based analogue of a standard, response-dependent concurrent VI VR, which we propose to call a concurrent variable-time variable ratio-time (concurrent VT VRT).

The list of intervals for both the interval and ratio schedules approximated a Poisson distribution, and the average interval for each list was varied through the course of the study. Table 1 lists the order and number of sessions for each schedule combination. In addition, Table 1 shows that the changeover-delay requirement was not used for the fifth condition and that the color assignment was switched during the sixth and seventh conditions. All sessions terminated after 60 reinforcers.

RESULTS

Figure 1 shows the relationship between time spent on each schedule and reinforcement rates for individual subjects in each of the seven conditions of the study. The graphs correspond to Equation 1, the linear form of the generalized matching law (Baum, 1973): The x axes give the logarithms of the reinforcement ratios, and the y axes give the logarithms of the time ratios. The parameters were estimated by the method of least squares and are plotted as solid lines; the dashed lines are for normative matching, that is, the special case where a and $\log b$ in Equation 1 equal 1.0 and 0, respectively. Table 2 lists the average amount of time spent at a schedule, the average number of reinforcers received from a schedule, and the average overall changeover rates for the last five sessions of each condition. Table 3 lists the matching law parameters and standard errors for each subject and the pooled results.

Figure 1 shows that the generalized matching law approximated the distribution of time spent on the alternatives for concurrent VT VRT. The equation accounted for between 91% and 98% of the variance in time ratios for individual subjects. The slopes of the fitted lines varied from 0.83 to 1.05, none of which differed significantly from 1.0 (see Table 3). The intercepts varied from −0.14 to +0.01, which likewise did not differ significantly from 0.0. In other words, the individual subjects' results were consistent with the narrower, normative version of the matching law.

Although the results for individual subjects were consistent with normative matching, the overall pattern of straight line fits did show a tendency for slopes of less than 1.0 and negative intercepts. This means that time ratios were less extreme than reinforcement ratios (undermatching), and that subjects tended to spend more time on the VRT schedule than predicted by reinforcement ratios (bias). Possibly these departures from simple matching represented influences that were systematic but too slight to detect with only seven observations per subject. To test this possibility, we fitted a line to the pooled results. Assuming that individual differences were not great, pooling should increase the power of the statistical tests.

The filled symbols of Figure 2 show the pooled results. (The open symbols are discussed below in the section on maximizing.) The slope (0.94) of the fitted line did not significantly differ from 1.0. However, the intercept, −0.065, did differ significantly from 0.0, albeit slightly (see Table 3).

When the schedule parameters are equal, matching of time allocation predicts that subjects should spend 100% (or nearly 100% if there is a COD) of the session on the VT

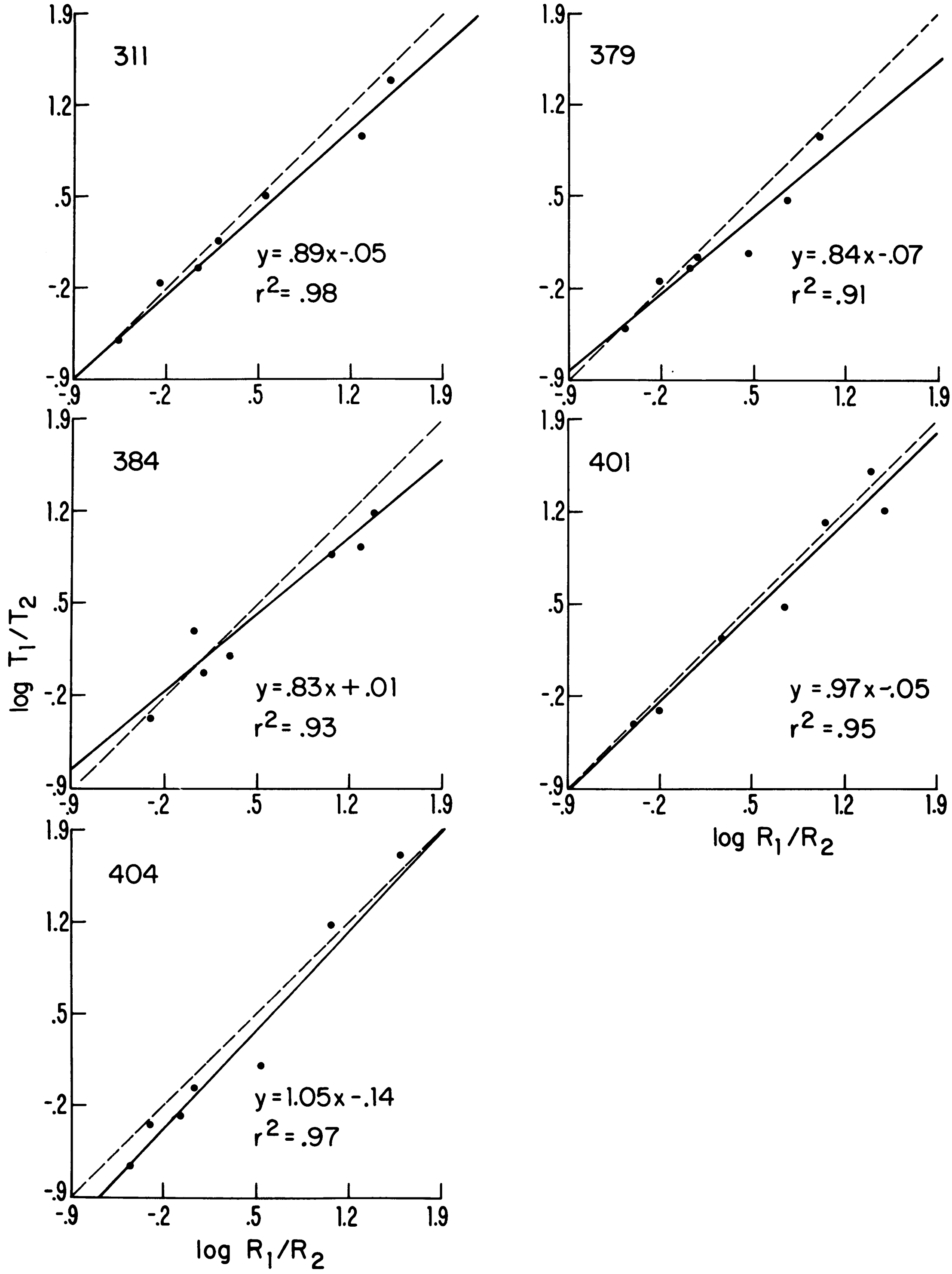

Fig. 1. Log ratio of time on VT to time on VRT as a function of log ratio of the obtained reinforcers, for each of the 5 subjects. Points are averages for the final five sessions on each of the seven experimental conditions. Dashed lines indicate values that correspond to normative matching; each solid line represents the best fitting linear equation, which is given along with variance accounted for.

schedule (see Equation B3 in Appendix B). In Conditions 2, 5, and 6, the VT and VRT schedules were each set at 30 s (see Table 1), and the median subject spent, respectively, 90%, 75%, and 95% of its time on VT. In Condition 5, the changeover delay was absent; Conditions 2 and 6 were the same except for the reversal of the color of the stimulus lights: red for the VT schedule in Condition 2 and green for the VT schedule in Condition 6.

DISCUSSION

Comparing Maximizing and Matching Predictions

For present purposes the essential difference between a matching account and a maximization account of choice in concurrent interval-ratio procedures is whether the local or the overall rates of reinforcement control behavior, where local reinforcement rate is R_i/T_i and overall reinforcement rate is $(R_1 + R_2)/(T_1 + T_2)$. For a matching account, local rates are decisive and overall rate is immaterial; for a maximization account, the reverse. On concurrent VI VR, response costs can affect both local and overall reinforcement rates, and because they are not directly measured, they are open to whatever interpretation favors one's theoretical predilection. The procedure used in this experiment neutralizes the response cost issue. Whatever the cost of being in the presence of a given houselight, it must have been the same for the two alternatives (except for color preference, which was checked); hence the distributions of time allocations that maximize reinforcement can be inferred from the observed overall reinforcement rates.

To evaluate the extent to which subjects approximated reinforcement maximization, we need a means of estimating how overall reinforcement rate depended on the allocation of time between VT and VRT. The relevant variables are (1) changeover rate, (2) the temporal pattern of switching between schedules, (3) overall time allocation to the two schedules, and (4) the schedules themselves as programmed. In Appendix A, these variables are represented mathematically, and the result is a model of the relationship between performance and reinforcement rate. This model was used to calculate (1) the time ratios that maximize reinforcement rate and (2) the time ratios that match their corresponding reinforcement ratios.

In Appendix A, Equation A11 is for the experimental conditions including the COD; Equation A8, for that omitting it. With the COD, it was assumed that the pigeon switched at a fixed *time* since the last switch. Without the COD, switching at a constant *rate*, independent of the time since the last switch, was assumed. Neither assumption precisely models actual performance; however, the two assumptions bracket observed changeover patterns (Heyman, 1982). As shown in Figure 3, the equations predict reasonably well the relationship between performance and reinforcement rate observed here.

Figure 3 provides an empirical test of the expected reinforcement rates formalized in Equations A8 and A11. On the x axis are the predicted reinforcement rates; on the y axis, the obtained reinforcement rates. The points with slashes through them are for Condition 5, lacking the COD, for which Equation A11 was used; all other points used Equation A8. The predictions account for 97% of the variance in the obtained reinforcement rates. The average difference between predicted and obtained rates was 4%. Within those limits, the equations correctly characterize the relationship between performance and overall reinforcement rate on concurrent VT VRT.

Figures 4 and 5 show the predicted relationship between proportion of time spent on VT and overall reinforcement rate for some of the schedules used here. The performances that maximized reinforcement rate were determined from Equations A8 and A11 by sweeping through the proportions of time spent on VT in 1% steps and solving for overall reinforcement rate at each value of time spent on VT. Maximizing distributions of time are represented as filled triangles.

The performances that matched relative response rate to relative reinforcement rate also were determined from Equations A8 and A11, and are plotted as open circles. For the case without the COD, $p/(1 - p)$ (i.e., the ratio of the proportion of time spent on VT to that spent on VRT) was set equal to R_1/R_2 (i.e., reinforcement rate on VT divided by that on VRT) and solved for p, which is the value of time spent on VT that conforms to matching. It is possible to calculate the matching value of p directly from the schedule parameters and

Table 2

Average amount of time spent at a schedule, average number of reinforcers received from a schedule, and average number of changeovers per minute for the last five sessions of each condition. Numbers within parentheses are standard deviations.

Subject	Schedule	Time (min)		Reinforcers		Changeovers
		VT	VRT	VT	VRT	
311	VT 50	15.8	11.4	37.1	22.0	13.2
	VRT 30	(1.6)	(0.3)	(1.6)	(1.5)	(1.3)
	VT 30	17.7	5.7	47.3	13.0	9.2
	VRT 30	(0.5)	(0.8)	(5.5)	(5.5)	(1.3)
	VT 50	3.1	12.1	13.3	46.8	4.5
	VRT 15	(0.5)	(0.9)	(1.3)	(3.4)	(0.6)
	VT 50	13.6	15.1	32.0	27.9	8.3
	VRT 30	(1.4)	(1.2)	(2.2)	(2.2)	(2.1)
	[a]VT 30	25.3	2.8	56.9	3.0	7.1
	VRT 30	(1.0)	(0.6)	(0.9)	(1.2)	(1.3)
	VT 30	28.3	1.2	58.4	1.6	1.6
	VRT 30	(1.0)	(0.7)	(0.9)	(1.1)	(0.7)
	VT 100	15.1	21.6	22.3	37.7	18.9
	VRT 30	(1.7)	(0.6)	(0.8)	(0.7)	(0.9)
379	VT 50	10.6	14.0	30.7	29.5	7.4
	VRT 30	(1.5)	(0.4)	(2.6)	(2.3)	(1.9)
	VT 30	24.6	2.8	54.4	5.7	2.0
	VRT 30	(2.2)	(1.1)	(2.1)	(2.8)	(1.2)
	VT 50	3.4	10.9	15.3	44.8	6.0
	VRT 15	(0.5)	(0.4)	(2.2)	(2.2)	(0.6)
	VT 50	14.3	14.3	32.5	27.7	11.2
	VRT 30	(1.2)	(1.6)	(2.1)	(2.0)	(1.5)
	[a]VT 30	11.9	10.2	44.6	15.5	20.8
	VRT 30	(0.7)	(0.4)	(3.1)	(1.2)	(0.9)
	VT 30	19.5	6.5	50.8	9.3	10.6
	VRT 30	(0.6)	(0.3)	(1.0)	(1.0)	(0.5)
	VT 100	15.7	21.6	22.9	37.1	17.3
	VRT 30	(1.3)	(0.4)	(0.7)	(0.6)	(1.8)
384	VT 50	12.5	13.7	32.9	26.7	5.8
	VRT 30	(1.9)	(1.0)	(2.1)	(1.8)	(1.5)
	VT 30	23.7	3.1	54.9	5.1	1.6
	VRT 30	(2.5)	(1.2)	(2.6)	(2.8)	(0.9)
	VT 50	5.3	11.6	20.2	39.9	3.7
	VRT 15	(0.7)	(0.7)	(2.9)	(2.8)	(0.5)
	VT 50	22.8	11.4	40.2	20.0	5.5
	VRT 30	(5.9)	(1.9)	(3.9)	(4.0)	(0.9)
	[a]VT 30	25.2	3.0	56.9	3.2	6.6
	VRT 30	(0.8)	(0.3)	(0.6)	(0.5)	(0.6)
	VT 30	28.2	1.4	57.9	2.2	1.3
	VRT 30	(1.1)	(0.3)	(1.9)	(1.8)	(0.3)
	VT 100	33.3	16.8	31.0	29.1	10.3
	VRT 30	(4.9)	(1.5)	(2.1)	(2.3)	(2.0)
401	VT 50	40.9	2.9	55.3	4.8	2.0
	VRT 30	(2.7)	(1.5)	(3.6)	(3.6)	(0.6)
	VT 30	28.5	0.9	57.8	2.4	0.3
	VRT 30	(0.5)	(0.4)	(1.6)	(0.8)	(0.4)
	VT 50	4.5	11.3	17.1	43.0	9.6
	VRT 15	(0.5)	(0.5)	(1.9)	(1.9)	(1.8)
	VT 50	21.6	11.8	39.2	20.8	6.9
	VRT 30	(5.6)	(2.0)	(4.7)	(4.8)	(2.3)
	[a]VT 30	18.6	6.4	50.0	10.1	22.0
	VRT 30	(0.5)	(0.5)	(2.6)	(3.1)	(1.9)
	VT 30	27.9	1.7	58.3	1.8	2.5
	VRT 30	(0.7)	(0.3)	(0.8)	(0.7)	(0.3)
	VT 100	12.6	24.4	22.8	37.2	37.4
	VRT 30	(0.6)	(1.3)	(0.7)	(0.8)	(5.5)

Table 2 (*Continued*)

		Time (min)		Reinforcers		
Subject	Schedule	VT	VRT	VT	VRT	Changeovers
404	VT 50	8.4	15.4	28.3	32.7	7.5
	VRT 30	(1.3)	(1.2)	(1.6)	(1.6)	(1.8)
	VT 30	25.7	2.1	55.6	4.6	0.8
	VRT 30	(1.2)	(0.8)	(1.7)	(2.2)	(0.2)
	VT 50	2.5	11.5	15.6	44.6	5.0
	VRT 15	(0.5)	(0.3)	(2.0)	(2.1)	(0.6)
	VT 50	13.4	15.4	31.2	28.8	4.3
	VRT 30	(3.5)	(1.9)	(3.8)	(3.7)	(0.7)
	[a]VT 30	12.5	10.1	45.9	14.0	33.8
	VRT 30	(0.9)	(0.5)	(0.4)	(1.1)	(3.8)
	VT 30	29.3	0.7	59.6	0.5	0.9
	VRT 30	(0.5)	(0.3)	(0.8)	(0.7)	(0.3)
	VT 100	11.0	23.0	20.5	39.5	20.8
	VRT 30	(2.5)	(0.9)	(1.1)	(1.0)	(3.6)

[a] COD omitted.

changeover rates without knowing the obtained values of R_1 and R_2 (see Appendix B for details). With the COD, the resulting equation does not have an analytic solution, so the matching value of p was estimated by testing all values in 1% steps and choosing the one that most closely matched the corresponding ratio of reinforcement rates. The values of p obtained in this way turned out to be within 2% of the values determined analytically from the equation for the procedure lacking the COD. That is to say, nearly identical matching solutions were predicted by the equation that included a representation of the COD and the one that did not. This implies that neither the COD nor the different patterns of switching correlated with each model had a major effect on reinforcement rate. Matching at the extrema—at 0 or 1.0 on the x axis—is not circled unless it is a meaningful solution to one or the other of the equations used to generate the curves in Figures 4 and 5.

Table 3

Matching law parameters[a] and (standard errors).

	Observed			Maximization		
Subject	Intercept (log *b*)	Slope (*a*)	Variance accounted for	Intercept (log *b*)	Slope (*a*)	Variance accounted for
311	−.050	.891	.984	−.660*	.855	.766
	(.041)	(.051)		(.055)	(.212)	
379	−.074	.843	.914	−.610*	.969	.792
	(.063)	(.116)		(.059)	(.222)	
384	.009	.825	.933	−.530*	.878	.757
	(.082)	(.099)		(.061)	(.222)	
401	−.049	.965	.952	−.550*	1.196	.745
	(.091)	(.097)		(.079)	(.312)	
404	−.140	1.050	.969	−.560*	.988	.736
	(.068)	(.085)		(.073)	(.271)	
Pooled	−.065*	.942	.955	−.584*	.964	.722
	(.028)	(.036)		(.039)	(.104)	

* significantly different from 0.0 with $p \leq .05$.

[a] For $\log\left[\frac{T_1}{T_2}\right] = a \log\left[\frac{R_1}{R_2}\right] + \log b$.

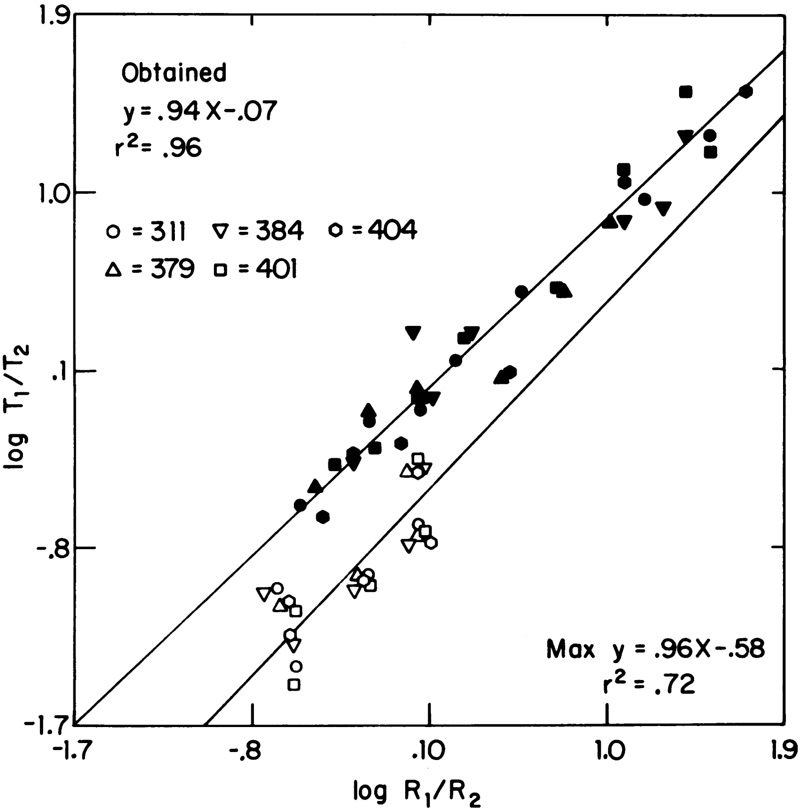

Fig. 2. Filled symbols replot the data points of Figure 1, with individual subjects identified; the line represents the best fitting linear equation for the pooled results. Open symbols show the time allocations and reinforcement ratios that would have maximized overall reinforcement rate, with the best fitting linear equation through these reinforcement maximizing points. Some of the maximizing points are not visible because they are superimposed on each other.

The three curves in each panel of Figures 4 and 5 correspond to different changeover rates, as quantified by I (for "inertia"; see Heyman, 1977). This parameter is the reciprocal of the sum of the two local changeover rates (i.e., the rates going one way or the other weighted by the time available for switching: $I = 1/(u_1 + u_2)$, where u_i is the local rate of switching from schedule i). For example, if I is equal to 1 s and the proportion of time spent on VT is .5, then the two local changeover rates sum to 30 per min ($u_2/(u_1 + u_2) = .5$ and $1/(u_1 + u_2) = 1$ s, so that changeover rate is equal to: .5(.5/s) + .5(.5/s); whereas if I is 100 s and the proportion of time spent on the VT remains .5, then the two local changeover rates sum to an overall rate of 0.3 per min, .5(.005/s) + .5(.005/s). The larger the value of I, the lower the average rate of switching. Evidence suggests that I increases approximately as a linear function of the COD duration (Heyman, 1982). For typical COD durations, pigeons switch at rates corresponding to I values of 3 to 10 s.

Figure 4 (a and b, without and with the COD, respectively) shows the matching and maximizing performances when VT and VRT schedules have equal time values. Maximization requires time allocations that favor the VRT (abscissa values less than .5), as is generally true of concurrent interval-ratio schedules. Matching, in contrast, can be satisfied by exclusive preference for the VT, if there is no COD, and by nearly exclusive preference, if there is one. Exclusive preference for VT when the two schedules have equal time values is not only nonoptimal for reinforcement, it yields the lowest possible rate of reinforcement. Yet, as Appendix B shows, this is the performance that is predicted by Equations A8 and A11. It is also the performance predicted by Myerson and Miezin's (1980) ki-

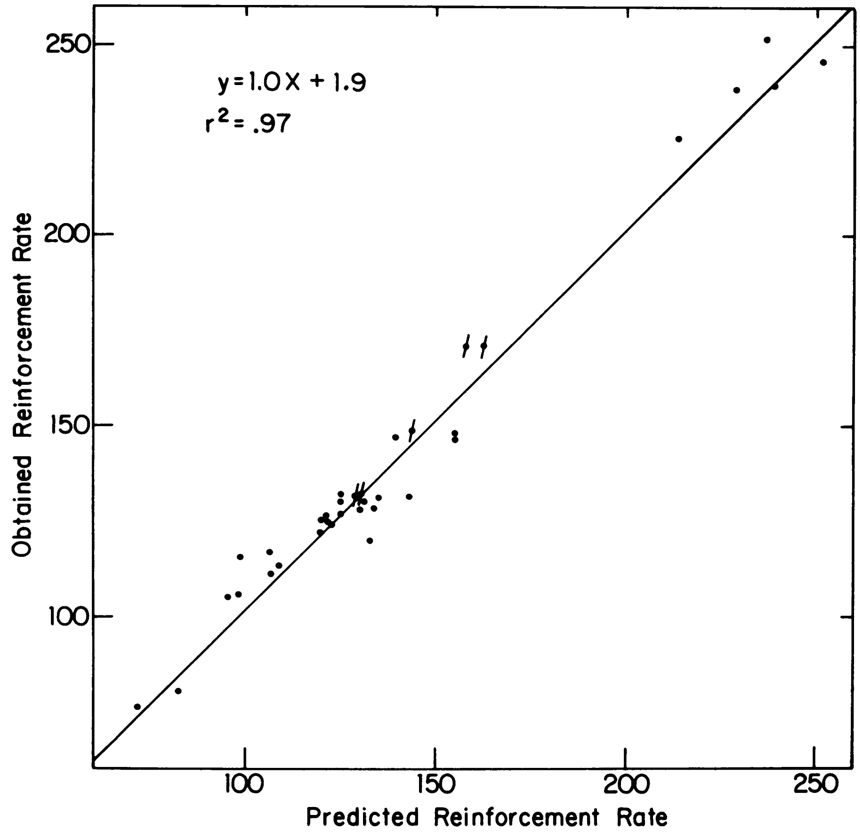

Fig. 3. From the derivations in Appendix A, reinforcement rates were predicted for each of the seven experimental conditions. Obtained reinforcement rates are plotted against these predictions, with the best fitting linear equation as shown. There are 35 points in all, seven conditions for each of 5 subjects. Points with slashes through them are for Condition 5, which lacked a COD.

netic model of switching and by melioration (Herrnstein & Vaughan, 1980). This ostensibly paradoxical outcome can be squared with intuition as follows: The only way a subject can match is either to cease responding at one alternative or to distribute its time at the two alternatives so that the local rate of reinforcement on VT is equal to that on VRT. Inasmuch as the schedule values are equal, any time spent on VRT results in a higher local reinforcement rate on VT because the VT timer can set up a reinforcer while the subject is attending VRT. (Equations A8 and A11 formalize this contingency in terms of programmed reinforcement rates.) Consequently, any time spent on VRT drives the local reinforcement rates apart (with VT > VRT) and any time spent on VT drives the local reinforcement rates toward equality. By the principle of melioration (Herrnstein & Vaughan), equilibrium is thus reached at or (with a COD) near exclusive preference for VT. This also happens to be the allocation of time that earns the minimum reinforcement rate (see Figure 4).

Conditions 2, 5, and 6 examined this unusual situation and, on each occasion, responding strongly favored VT (90%, 75%, and 95%, respectively, for the three conditions). The lowest of the three, although still far from reinforcement maximizing, may have deviated from matching as much as it did because it lacked the COD. This may follow from melioration itself, inasmuch as absence of the COD allows the two alternatives to lose their separate identities as response categories (see Herrnstein, 1982, for melioration in relation to response categories).

Figure 5 (upper and lower panels with and without the COD, respectively) shows the matching and maximizing performances when the time values for VT and VRT are unequal.

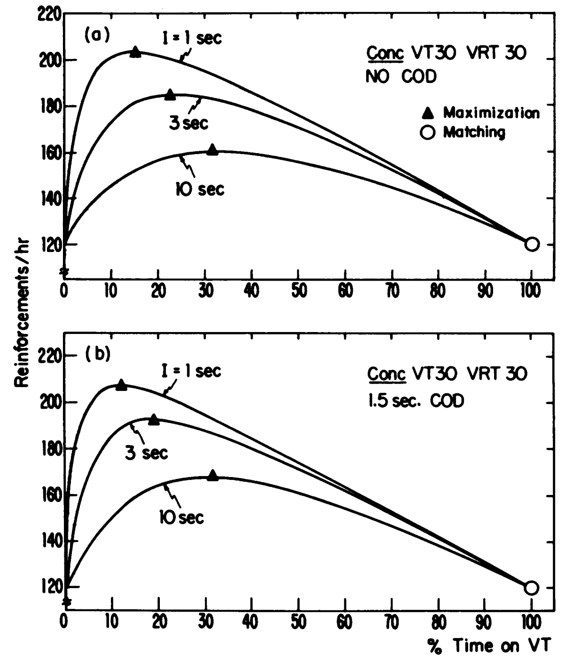

Fig. 4. Overall reinforcement rates as functions of time allocation, for three different changeover rates (higher values of *I* represent lower changeover rates), on concurrent VT 30 VRT 30. Filled triangles are reinforcement maximization points; open circles, matching points. See text and appendices for further explanation. Panel a: no COD; panel b: 1.5-s COD.

Here, as in Figure 4, neither COD nor the assumed pattern (as distinguished from the rate) of switching much affected reinforcement rates. Reinforcement maximization again calls for preference for VRT (abscissa values below 0.5). The matching points vary with changeover rates; for the rates typically observed (i.e., *I* < 10 s), matching implies preference for VT (abscissa values above 0.5).

The same schedule has different matching solutions because of the link between changeover rate and local reinforcement rate for the VT schedule (see Herrnstein & Vaughan, 1980, for an account of this link with respect to VI schedules). The higher the changeover rate, the greater the local reinforcement rate on VT (or VI). Consequently, as changeover rate increases (or, equivalently, as *I* decreases), the matching performance shifts toward VT. However, as Figure 5 shows, different matching performances yield exactly the same overall reinforcement rate (a relationship explicated for concurrent VI VR in Prelec & Herrnstein, 1978). For a subject that matches, the overall reinforcement rate is in fact predetermined by the schedule value set for VRT.

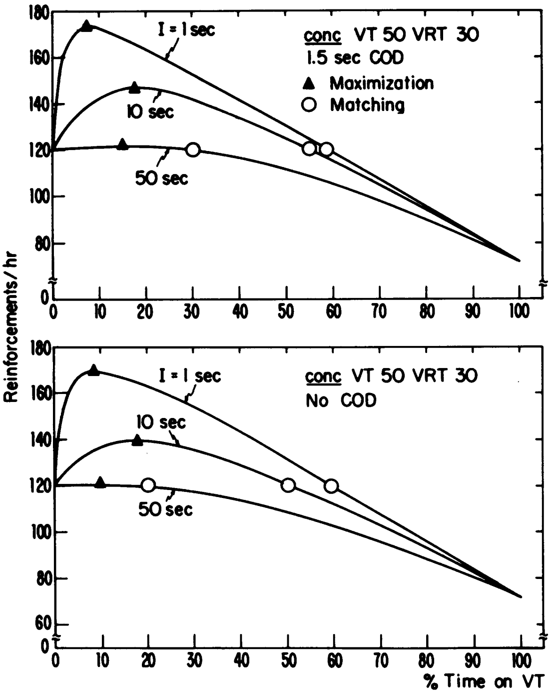

Fig. 5. Same as Figure 4, for concurrent VT 50 VRT 30. Upper panel: 1.5-s COD; lower panel: no COD.

Figure 2 provides a comparison of the observed preferences of subjects and the preferences predicted by maximization. The open symbols of Figure 2 show, as a function of their corresponding reinforcement ratios, the maximizing performances, which invariably have a *y*-ordinate value of less than 0.0. In other words, to maximize reinforcement, the subjects should have spent more than 50% of every session under every condition on the VRT schedules. Intuitively, it is obvious why this is so. While the subject is at the VRT alternative, reinforcement can be programmed on the VT. Time spent on VRT brings reinforcement on both schedules nearer. But time spent at the VT alternative accumulates no time for the VRT. To maximize reinforcement, the subject should then spend most of its time on VRT, visiting VT only occasionally and briefly, so as to collect reinforcers that are due. The median maximizing prediction, based on Equations A8 and A11, was about an 80% time allocation for the VRT, with a range of about 65% to 92%. None of the sub-

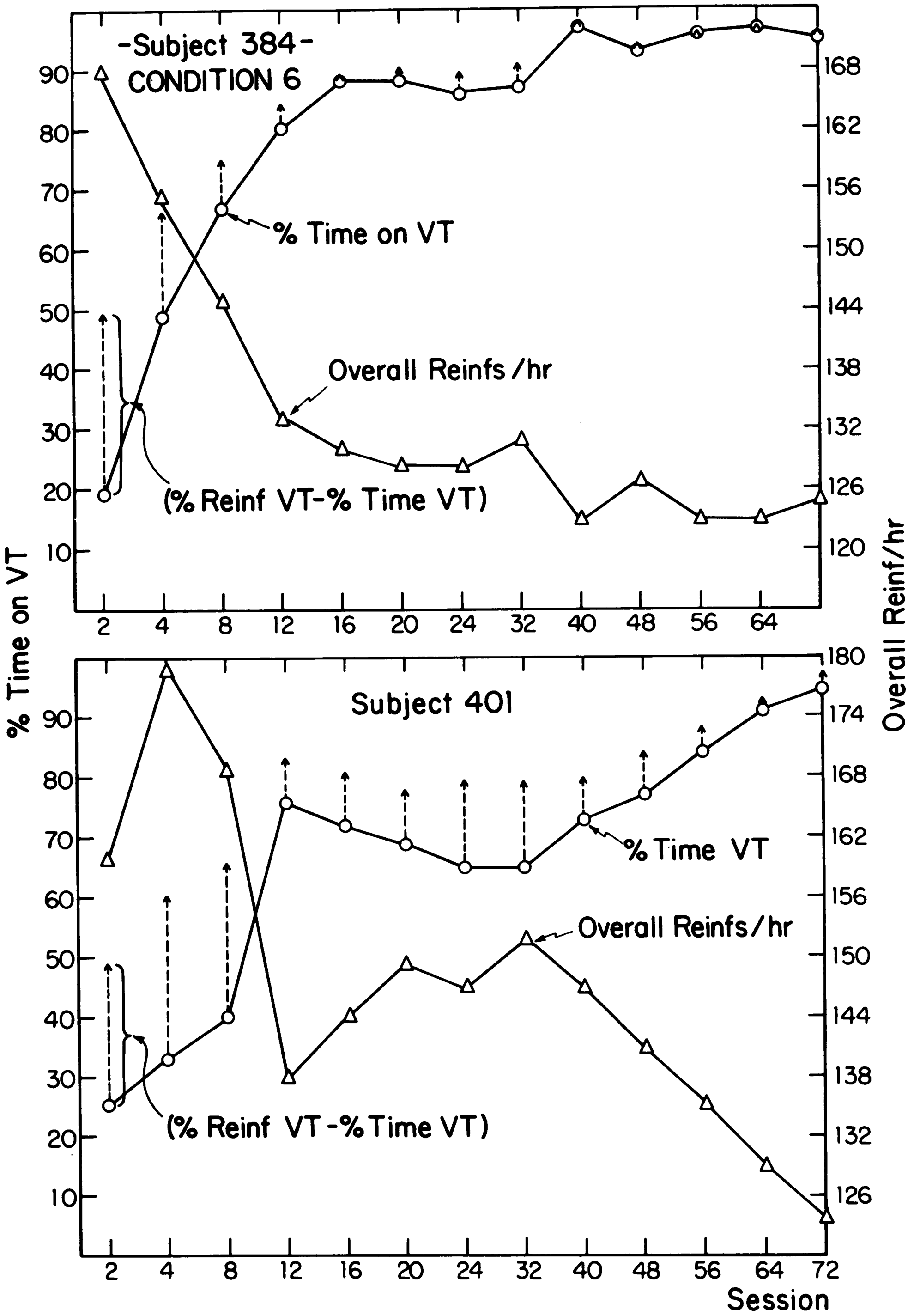

Fig. 6. Transition to concurrent VT 30 VRT 30 (Condition 6) for two pigeons. Graphs show percentage of time spent on VT (circles), overall rates of reinforcement (triangles), and the percentage of reinforcement on VT minus percentage of time on VT (tips of arrows), for consecutive blocks of varying numbers of sessions. The longer the arrows, the greater the deviation from matching. As the deviation from matching contracted, the overall rate of reinforcement fell.

Table 4

Median obtained, maximum, and $p = .50$ reinforcement rates.

Condition	Maximum	$p = .50$	Obtained
1	159	128	133
2	166	161	132
3	270	189	237
4	159	128	125
5	198	175	144
6	192	172	122
7	145	95	98

jects even approximated this invariant preference for VRT. (These estimates are based on each subject's observed changeover rates. If the estimation procedure allowed those rates to vary freely, the predicted value for VRT at maximization would be even more extreme.)

If the pigeons had been maximizing, the best fitting generalized matching law line in Figure 2 would have accounted for 72% of the variance in performance, and would have a slope of 0.96 and an intercept of −0.58. The slopes of the maximizing performances and the obtained time ratios, then, were about the same—0.96 and 0.94, respectively—but the intercepts differed markedly (by 19 standard errors of the observed data, see Table 3). The variances accounted for also differed—72% for the maximizing line, 96% for the observed best fitting line—which indicates that maximizing dictated less conformity to any straight line in the log-ratio plane than the pigeons actually displayed.

The irrelevance of overall reinforcement rate is nowhere more obvious than for Conditions 2, 5, and 6. Here the performances shifted toward matching even though the schedule parameters had been changed so that the time ratios from the just previous condition approximated the maximizing solution for the new condition (see Table 2). The shift toward matching therefore reduced overall reinforcement rate. For example, Figure 6 shows how 2 pigeons tracked the matching relationship in Condition 6, and how the overall rate of reinforcement fell as they did so. The maximizing performance would have been approximately an 87% time allocation to VRT. The stimulus lights had just been switched, so that in the first two sessions of the new condition, the subjects were allocating time in proportions that approximated maximization. However, matching, as calculated from the feedback equations in Appendix B, called for nearly a 100% time allocation to VT. Both subjects shifted toward extreme preference for VT, significantly reducing overall reinforcement as they did so.

The dashed arrows rising from the data points in Figure 6 show the extent of the difference between the percentage of reinforcers received from the VT and the percentage of time spent there, for consecutive blocks of sessions. This difference is also the deviation from matching. The deviation was large at the start of the condition, but decreased as preference shifted toward the VT. The figure indicates that the feedback relations of the schedule did, in fact, result in a reduction in overall rate of reinforcement as matching was approximated. Whenever a pigeon transiently reversed the trend toward lower overall reinforcement rates, the deviation from matching grew, and vice versa. The performance clearly shifted so as to reduce the deviation from matching.

Table 4 lists the obtained reinforcement rates, the reinforcement rates for a hypothetical subject maximizing reinforcement rates, and the reinforcement rates for another hypothetical subject that spends 50% of its time at each schedule, switching at random and without regard to the reinforcement contingencies. By this measure, the observed pigeons were outperformed by both the hypothetical maximizer and the hypothetical contingency-ignorer, by averages of 42 and 8 reinforcers per hour, respectively. In matching as well as they did, the pigeons earned fewer reinforcers than they could have, not only by maximizing, but by ignoring the contingencies of reinforcement altogether.

The approximately successful prediction that the subjects should spend most or all of the session time on the VT schedule when the time requirements are equal has three implications. First, inasmuch as the prediction was based on the logical properties of the matching law and on the feedback relations expressed by Equations A8 and A11, both the law and the modeling approach exemplified by the equations gain some further degree of confirmation in a new situation. Second, exclusive preference for the VT produces a reinforcement rate as low as any possible in the situation, so that from the point of view of reinforcement maximization, matching is the least likely possible outcome. This argues against maximization in a procedure designed to eliminate the need for estimating response costs. Third, exclusive preference for VRT, rather than the observed preference for the VT, would also have satisfied the matching law, but it would have violated the principle of melioration or Myerson and Miezin's (1980) kinetic theory. The results strengthen the case for melioration or some other dynamic principle that makes matching the equilibrium state for time allocation.

Other Findings

Since Herrnstein and Heyman's (1979) article appeared, several studies have tested its claim to have challenged the notion of overall reinforcement maximization. We will here review five of them, three pro and two con, and then attempt to draw a unifying conclusion.

1. As mentioned in the introduction, DeCarlo (1985) reported an experiment similar in all respects to the present one—namely, an examination of choice between ratio and interval alternatives in the absence of a discrete response. For data pooled across his 4 pigeons, the generalized matching law accounted for 96.7% of the variance in choice proportions, and the slope and intercept were 0.920 and −0.062, respectively. These values closely match our pooled results: 95.5% variance accounted for, and slope and intercept of 0.940 and −0.065, respectively. The tendency to match, DeCarlo estimated, was costing his pigeons an average 20 reinforcers per hour across experimental conditions. In one condition, VT and VRT durations were the same (45 s). According to the feedback equations (Appendix B), VT should have been preferred almost exclusively. The 4 pigeons spent 94%, 99%, 95%, and 56% of the time on VT. In the present study, the median preference under comparable conditions was 90%.

2. Houston (1986; see also Davies & Houston, 1983) analyzed the field behavior of pied wagtails, an insectivorous bird native to the region around Oxford University. During the winter, some of the birds feed alternately in territories they defend against other feeders and in the flock along with other wagtails. Houston showed that the feeding schedule in the flock approximates a variable-ratio schedule with different ratios from day to day, while that in the territory approximates a variable interval inasmuch as the probability of finding prey rises as a negatively accelerated function of time since the last visit to a particular stretch of ground. Together, the two alternatives constitute an approximate concurrent VI VR. Over several winters Davies kept track of the time spent by individual wagtails in the flock and in the territory, and of the number of prey taken in each. Using the present Equation 1, Houston reported that the wagtails undermatched (exponent equal to 0.78) and had a bias toward the territory (the interval alternative) of 1.8. The equation accounted for 94% of the variance in time allocations. As Houston pointed out, the wagtails did not maximize overall feeding rate; to do so on concurrent VI VR requires a bias toward the ratio alternative.

3. A different approach to interval-ratio choices has been taken by Williams (1985). In a discrete-trial procedure, 4 rats chose between a pair of alternatives for which reinforcement was programmed by variable-ratio and variable-interval schedules. A single depression of either lever terminated the trial and, depending on the two reinforcement schedules, either did or did not deliver a reinforcer. For the variable-ratio alternative, the probability of reinforcement was a constant fraction of the number of times that that alternative was chosen. For the variable-interval alternative, reinforcement was programmed by the passage of time during intertrial intervals, and thus depended on the number of trials since the last choice of that alternative. Ten pairs of schedule values were examined.

The logic of the experiment relies, first of all, on its use of a discrete-trial procedure, which obviates the differential response cost in interval and ratio schedules that is postulated by global maximization theorists (Green et al., 1983; Rachlin et al., 1981). Moreover, for any given pair of schedule parameters, it is possible to calculate exactly, for every pattern of consecutive choices of the two alternatives, which alternative has the higher probability of reinforcement on any given trial (de Villiers, 1977; Nevin, 1969). Williams could therefore test whether the rats were tracking local reinforcement probabilities, as required by a momentary or local maximizing theory (e.g., Hinson & Staddon, 1983; Shimp, 1969; Silberberg, Hamilton, Ziriax, & Casey, 1978).

Finally, Baum (1981) and Prelec (1982) have shown that on concurrent VI VR, reinforcement maximization implies different outcomes for parametric variations of the ratio and of the interval components, as follows: Varying the interval schedule value should produce a bias (*b* differing from 1.0 in Equation 1) toward the ratio equal to the square root of the invariant ratio-schedule value, and varying the ratio-schedule value should primarily produce undermatching (*a* less than 1.0 in Equation 1) in addition to bias. Williams chose schedule values so as to test this particular implication of global maximization.

Williams' results failed to support global, momentary, or local maximization. First, the size of the bias term was not dependent on the square root of the ratio requirement when the interval-schedule value was varied. Next, the pattern of switches seemed to have no relationship to the immediately higher probability of reinforcement on any given trial. Finally, the generalized matching law provided as good a fit to these discrete choices, for which problems of response cost are bypassed, as for the continuous choice paradigm used in earlier studies of concurrent VI VR (e.g., Herrnstein & Heyman, 1979). The rats undermatched, whereas our pigeons either did not or did so less, but it is variance accounted for and intercept that discriminate decisively between overall maximization and generalized matching, and, for those two variables, Williams' data strongly support matching as opposed to global maximization.

4. Green, Rachlin, and Hanson (1983) report results that fall about midway between matching and maximizing predictions for a procedure that was ostensibly similar to concurrent VI VR. Pigeons could peck a left or a right key. A peck on the left key changed that key's color from amber to red for 1 s, during which time pecking was reinforced on a variable-ratio schedule. A peck on the right key changed that key's color from amber to green for 1 s, during which time pecks were reinforced according to a nonconventional VR schedule: A prescribed number of pecks produced reinforcement, as is the case for VR, but in contrast with the typical arrangement, pecks on either key counted toward reinforcement. For example, if the subject fulfilled the right-key ratio requirement with responses on the left key, the right-key reinforcer was saved and delivered when the subject switched to the right key. This schedule, then, was a response-count-driven analogue of an interval schedule.

Green et al. assumed that amber light measured time spent at leisure and that the other two lights (red and green) measured time spent responding. They did not test this assumption and there is reason, within their own data, to believe that it is not entirely correct. If the 1-s periods of red and green were actually free of leisure, and if differential amounts of leisure account for the different local response rates on interval and ratio schedules (see introduction), then local response rates should have been the same during red and green periods. From data provided by Green (personal communication, June 1984), it is apparent that this was not the case.

Even though their procedure evidently did not quite fulfill the stated objective, it remains noteworthy that the subjects deviated more from matching than has been customary in interval-ratio procedures of various sorts. The discrepancy, we suggest, arises from the nontemporal nature of their procedure. In concurrent schedules involving VI or VT, reinforcement and its maximization depend on the passage of time, in particular the allocation of time across alternatives. In the procedure of Green et al., it depended only on the output of pecks, not time spent, or rate of, pecking. Therefore, merely waiting brought reinforcement no closer. Sequences of pecks had fixed reinforcement probabilities, independent of the passage of time. In such procedures, it has been observed before that subjects may learn response sequences directly, in effect learning a new response class that is a compound of the separate alternatives in a concurrent procedure (e.g., Herrnstein, 1958; Herrnstein & Loveland, 1975; Schwartz, 1980, 1981). Herrnstein (1982) has suggested that the learning of such new classes is itself attributable to melioration and is therefore no challenge to the theory. To the extent that new response classes have formed (which may, in fact, be difficult to assess quantitatively), the matching law is more appropriately tested in such procedures with the new response class than with the components of which it is composed. Matching is, however, found in concurrent procedures without the learning of particular response sequences (e.g., Heyman, 1979; Nevin, 1969; Silberberg et al., 1978).

5. Ziriax and Silberberg (1984) focused on a methodological aspect of concurrent VI VR. Because reinforcement rate is proportional to response rate on ratio schedules, reinforcement ratios, they note correctly, are more constrained by choice ratios on concurrent VI VR than on concurrent VI VI. Ziriax and Silberberg used various methods to investigate the dependency of reinforcement on choice and concluded that, on concurrent VI VR, matching reflects the behavior of the procedure, not the subject. As they say, on concurrent VI VR, "over a considerable range of choice ratios, matching-like data should be anticipated no matter what process governs choice" (pp. 90–91). They conclude: "Because reinforcer frequency follows choice on concurrent VI VR schedules, orderly matching functions in terms of these variables were largely preordained. For this reason, the use of concurrent VI VR schedules as a test among different accounts of matching relations is inappropriate" (p. 99).

The force of this argument depends on just how narrow, in fact, the constraint around matching is on interval-ratio procedures and how the constraint relates to reinforcement maximization. In Herrnstein and Heyman (1979), we presented the feasible choice proportions, given observed rates of responding and switching, that would have maximized reinforcement. At issue is whether these maximizing performances are necessarily close to, or even indistinguishable from, the observed matching. They were neither. Maximizing predicted significant undermatching—a slope of 0.72 in Equation 1. The observed slope was 1.04. Maximizing predicted an intercept of -0.38, biased toward the VR; the observed intercept was 0.11, biased toward the VI. The maximizing performance had a goodness-of-fit score (i.e., variance accounted for by the generalized matching law) of 0.64; the observed data, a goodness-of-fit score of 0.97.

Ziriax and Silberberg say that their investigation of concurrent VI VR was motivated by what they call the "surprising" observation that, in Herrnstein and Heyman's data, different subjects on the same schedule had different preferences, yet showed about equally good matching. This, they suggest, indicates that matching on concurrent VI VR reflects the procedure itself, rather than behavior. Their interpretation assumes that relative reinforcement rate for a given concurrent VI VR depends only on relative response rate. However, for this schedule, relative reinforcement rate also depends on absolute response rate and on changeover rates. The same concurrent VI VR can have many different matching solutions (see Figure 5 and accompanying discussion for the role of changeovers). For example, consider 2 subjects discussed by Ziriax and Silberberg. On concurrent VI 40 s VR 30 (Herrnstein & Heyman, 1979), the choice proportions (in pecks) for the VI were 0.93 for Pigeon 83 and 0.13 for Pigeon 365, yet both subjects were nearly matching (1 and 5 percentage-point deviations from matching, respectively). This difference is not, however, surprising, given the difference in their absolute response rates. If Pigeon 365 had responded on the VR at the same rate as Pigeon 83, it would have received 45%, rather than 18%, of its reinforcement from the VI and, thereby, have deviated from matching by 32 percentage points. Note also that this example illustrates again that substantial deviations from normative matching are possible on concurrent VI VR.

Ziriax and Silberberg explored analytically the range of possible choice and reinforcement relationships in the Herrnstein and Heyman (1979) study. Using either a reinforcement-feedback equation or a computer simulation, they generated a series of predicted reinforcement ratios by varying choice, local response rate, and changeover rate. However, the predictions were based on a single concurrent schedule (VI 30 s VR 30); Herrnstein and Heyman had used a series of schedule values, as is standard practice. Figure 7 indicates that a single schedule underestimates the range of possible outcomes on interval-ratio procedures.

Figure 7 shows a possible set of choice ratios and reinforcement ratios, given the schedule parameters and the observed switching rates in the present experiment. The choice ratios were determined by setting the changeover parameter (I) and schedule parameters in Equations A8 and A11 to the values of the present experiment, then varying time allocation (p) to find the value that differed most widely from its reinforcement allocation. For example, in the second condition of the ex-

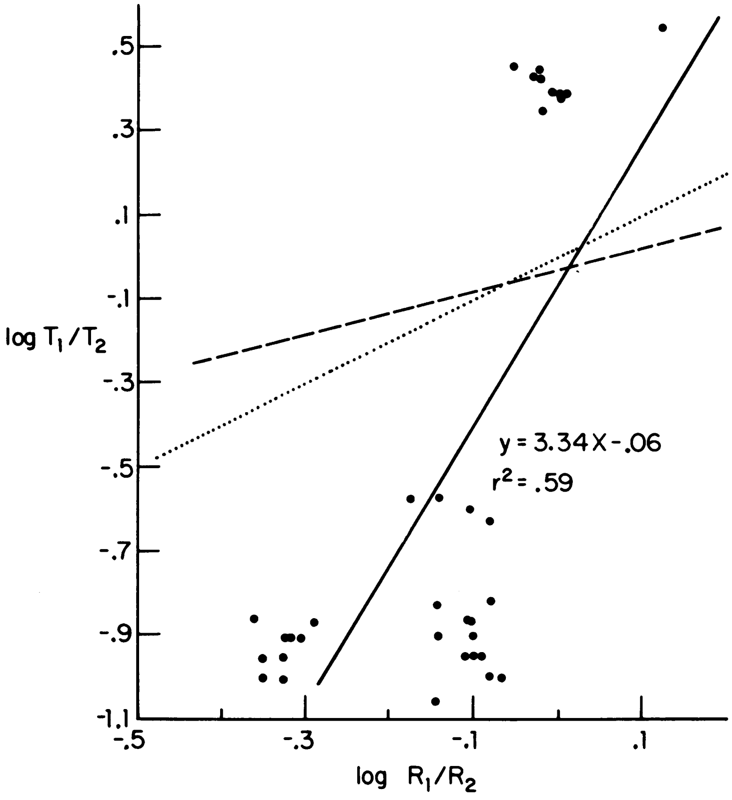

Fig. 7. Using the equations in Appendix A, the programmed schedule values, and the observed changeover rates for each subject under each condition, we calculated the maximum possible difference between log ratio of time allocations and log ratio of reinforcement rates. The solid line plots the best fitting linear equation for these 35 differences. The dashed line plots the best fitting equation for the observed differences; the dotted line, for perfect normative matching. This analysis evaluates the claim (Ziriax & Silberberg, 1984) that on interval-ratio schedules, subjects are constrained to match.

periment (VT 30 VRT 30), Pigeon 311, given its changeover rate, could have received 44% of its reinforcers on VT by spending 11% of its time there, a 33 percentage-point deviation from normative matching. The solid line in Figure 7 is the best fitting straight line relating choice and reinforcement ratios for this selection of maximal discrepancies between the variables. It has a slope of 3.34 and an intercept of −0.06. Note also that the variance accounted for is only 59%, for this shows that performance on an interval-ratio procedure may be ill described by *any* fit of the generalized matching law. For purposes of comparison, the dashed line indicates the results actually obtained, and the dotted line, those for normative matching.

Recall that the function fitted to the feasible maximizing performance in Herrnstein and Heyman (1979) had a slope of 0.72 and an intercept of −0.38. Variance accounted for by the maximizing prediction was about 60%. The relationship between choice and reinforcement ratios on concurrent interval-ratio procedures has therefore been shown to permit a range of variation encompassing at least a four-fold spread of slopes and a six-fold spread of intercepts in Equation 1, as well as variances not much accounted for by the equation. Inasmuch as this is a far greater range than has been observed for concurrent VI VI, it is not correct that the matching observed for interval-ratio procedures is necessarily any more a result of procedural constraints than it is for interval-interval procedures.

Ziriax and Silberberg's own data provided direct evidence favoring matching over maximization in the one condition of their experiment in which such a direct comparison was possible. The schedule was concurrent VI 30 s

VR 30, and the pigeons' choice ratios were allowed to vary freely. Maximization predicts a preference for VR. From Equation B4 (Appendix B) and the schedule parameters and local response rates published by Ziriax and Silberberg, it follows that matching predicts a time allocation of 0.75 to the VI. The observed median time allocation was 0.83, which deviated from matching in the direction opposite to that implied by maximization.

In short, Ziriax and Silberberg correctly note that on interval-ratio procedures, reinforcement ratios track choice ratios more closely than on interval-interval procedures. They fail to note three additional facts that vitiate much of the relevance of the tracking. First, the procedural constraint is far looser than the observed variances accounted for by Equation 1 on interval-ratio procedures. Second, the constraint is not nearly as close to normative matching as has been observed on interval-ratio procedures. Third, the constraint has no detectible bearing on the observed failure of subjects to approximate the maximum feasible reinforcement rate on these procedures.

Conclusions

Choice on interval-ratio concurrent procedures has repeatedly conformed to the matching law to a close approximation (e.g., Bacotti, 1977; DeCarlo, 1985; Herrnstein, 1970; Herrnstein & Heyman, 1979; Williams, 1985). The special significance of matching with this class of procedures is that it usually exacts a considerable cost in overall reinforcement (see also Houston, 1983, for an even larger estimate of lost reinforcers than ours). Maximization theorists have countered by suggesting either that the matching is an incidental by-product of response costs or that interval-ratio schedules force subjects to match because of the direct effect of ratio responding on rate of ratio reinforcement. The present results, as well as other findings reviewed above, demonstrate that response costs explain neither the observed deviations from reinforcement maximization nor the matching that occurs. In addition, it is easy to show that interval-ratio procedures do not force the observed matching. A subject working on an interval-ratio procedure may deviate widely from matching and may obtain additional reinforcers by doing so. From a wide range of possible performances, subjects approximate normative matching and thereby often fail to obtain large proportions of the available reinforcers.

On interval-ratio schedules, the subject can usually gain more reinforcers by not matching and by instead concentrating on the ratio alternative, visiting the interval alternative only briefly to collect the reinforcers set up in its absence. Consequently, in order to maximize total reinforcement, subjects must receive unequal local reinforcement rates at the competing alternatives. They should focus on the "bottom line," which is to say, on the gains (and losses, if any) aggregated across all alternatives. Although the long-term bottom line is an essential feature of global maximization theory and rationalistic accounts of behavior, such as that of evolutionary biology and neoclassical economics (see Vaughan & Herrnstein, in press, for discussion), rats and pigeons fail to maximize it when it conflicts with matching. They often maximize it when it does not conflict. The time has come to accept the data and move beyond them, to explore further the empirical and theoretical implications of the limited behavioral adaptation implicit in matching.

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APPENDIX A

The purpose of Appendix A is to describe the logical basis of the matching and maximizing predictions in Figures 4 and 5. We first calculate the moment-to-moment reinforcement rates, then the expected overall reinforcement rates.

When the VRT timer was off, a changeover-key response turned it on, and, conversely, when the VRT timer was on, a changeover-key response turned it off. When the timer was on, it provided reinforcement at the scheduled moment-to-moment rate, which was simply:

$$1/\mathrm{VRT}. \qquad \text{(A1)}$$

The VT timer ran until it set up a reinforcer. If the subject was at the VT side, the reinforcer was delivered immediately and the timer was restarted. If the subject was at the VRT, the reinforcer was held until the subject returned to the VT. Reinforcement on the VT thus depended on the frequency of switches to the VT, the frequency of switches to the VRT, and the temporal pattern of switches.

In principle, the temporal pattern represents the most difficult modeling problem, because there are unlimited possible patterns and a period of continuous behavior is likely to consist of a mixture of some of the many possibilities. Fortunately, the problem proves to be less intractable than it may seem. First, it is possible to calculate expected reinforcement rates for the switching patterns producing the highest (Heyman, 1977; Houston & McNamara, 1981) and lowest (Heyman, 1982) reinforcement rates. These two patterns bracket the expected reinforcement rate for any particular pattern or any mixture of patterns. Second, for typical schedule parameters, no particular switching pattern will compromise the validity of the model, because switching pattern has little effect on reinforcement rate. For example, the most and least efficient switching patterns produce reinforcement rates that differ by no more than 2% for typical schedule values and changeover rates (Heyman, 1982).

Consider a VT that does not involve a COD and has an exponential distribution of programmed interreinforcement times, as in the experiment described in this paper. While the subject was at the VT, the rate of obtaining a reinforcer was the same as the programmed rate because there was no response requirement. This rate is:

$$1/\mathrm{VT}. \qquad \text{(A2)}$$

While the subject was at the VRT, the probability that the VT would set up a reinforcer was:

$$1 - e^{-t/\mathrm{VT}}, \qquad \text{(A3)}$$

where t is the time since the last switch from the VT. For Poisson switching, which is the least efficient pattern, the rate of switching back to the VT at time t is:

$$u_2 e^{-u_2 t}, \qquad \text{(A4)}$$

where u_2 is the local rate of switching from VRT to VT. The expected probability that the subject receives a reinforcer at time t on the VT (following a switch) is therefore:

$$\int_0^t (1 - e)^{-t/\mathrm{VT}} u_2 e^{-u_2 t}\, dt = \frac{1}{1 + \mathrm{VT}u_2}, \qquad \text{(A5)}$$

where t is the time since the last switch *from* the VT.

Equations A1, A2, and A5 give the momentary reinforcement rates for the three different ways of producing reinforcers on concurrent VT VRT: on the VRT under any conditions, on the VT immediately after a switch to the VT, and on the VT during the rest of the visit there. To determine overall reinforcement rate, it is necessary only to supply estimates of the expected length of a visit to each schedule. For Poisson switching, expected visit duration is:

$$\int_0^\infty t u_i e^{u_i t}\, dt = \frac{1}{u_i}, \qquad \text{(A6)}$$

where u_i is the local rate of switching away from side i.

Combining Equations A1, A2, A5, and A6, and taking advantage of the fact that overall division of time between the two schedules is determined by the two local changeover rates—that is, $p = u_2/(u_1 + u_2)$—the expected overall reinforcement rate is:

$$E(R) = \frac{p}{\mathrm{VT}} + \frac{1 - p}{\mathrm{VT} + \dfrac{1}{u_2}} + \frac{1 - p}{\mathrm{VRT}}. \qquad \text{(A7)}$$

Each of the quotients in Equation A7 corresponds to one of the ways that reinforcement can occur in concurrent VT VRT. The first quotient says that while the subject is at the VT, reinforcement occurs at the rate set by the VT timer. This holds for p of the session time. The second quotient applies to reinforcers that are picked up upon a switch to the VT. This reinforcement rate is determined by the mean VT interval and the mean visit time to the VRT; it holds for $1 - p$ of the session time. The third quotient says that while the subject is at the VRT, reinforcement occurs at the rate set by the VRT timer.

For a more convenient graphic presentation of Equation A7, it helps to include the changeover constant, $I = 1/(u_1 + u_2)$. The result is:

$$E(R) = \frac{p}{\mathrm{VT}} + \frac{1-p}{\mathrm{VT} + \dfrac{I}{p}} + \frac{1-p}{\mathrm{VRT}}. \qquad \text{(A8)}$$

Note that $I/p = 1/u_2$ and that $I/(1 - p) = 1/u_1$. Figures 4a and 5a plot Equation A8.

Poisson switching yields the lowest reinforcement rate for a given value of p and I. In contrast, switching at a fixed time since the last switch yields the highest. (Heyman, 1977, and Houston & McNamara, 1981, independently derived the feedback function for fixed-time switching; Houston & McNamara proved that it is the most efficient pattern.)

For fixed-time switching, the probability that a reinforcer has set up at the VT while the subject was at the VRT is:

$$1 - e^{-T_2/\mathrm{VT}}, \qquad \text{(A9)}$$

where T_2 is the duration of the visit to the VRT. While the subject is at VT or VRT, reinforcement occurs at the programmed rates, as with Poisson switching. The overall expected reinforcement rate for fixed-time switching is therefore:

$$E(R) = \frac{T_1}{(T_1 + T_2)\mathrm{VT}} + \frac{1 - e^{-T_2/\mathrm{VT}}}{(T_1 + T_2)} + \frac{T_2}{(T_1 + T_2)\mathrm{VRT}}. \qquad \text{(A10)}$$

For fixed-time switching, it is possible to represent the effects of the changeover COD on reinforcement rate. The result, first pointed out by Houston and McNamara, is:

$$E(R) = \frac{T_1 - C}{(T_1 + T_2)\mathrm{VT}} + \frac{1 - e^{(-T_2+C)/\mathrm{VT}}}{T_1 + T_2} + \frac{T_2}{(T_1 + T_2)\mathrm{VRT}}, \qquad \text{(A11)}$$

where C equals the duration of the COD. Figures 4b and 5b plot Equation A11. (Note that $\mathrm{I} = 1/[1/T_1 + 1/T_2]$.)

APPENDIX B

The purpose of Appendix B is to show how to calculate the allocation of time that satisfies normative (fixed-parameter) matching from the scheduled reinforcement rates on concurrent VT VRT, and also the corollary calculation for concurrent VI VR.

The matching relationship is:

$$\frac{p}{1-p} = \frac{R_1}{R_2}. \tag{B1}$$

From Equation A8, the feedback function for Poisson switching, it is possible to calculate R_1 and R_2. Substitution gives:

$$\frac{p}{1-p} = \frac{\dfrac{p}{\mathrm{VT}} + \dfrac{1-p}{\mathrm{VT} + \dfrac{I}{p}}}{\dfrac{1-p}{\mathrm{VRT}}}. \tag{B2}$$

Rearranging to solve for p:

$$p = \frac{\mathrm{VT\ VRT} + I(\mathrm{VRT} - \mathrm{VT})}{\mathrm{VT} \cdot \mathrm{VT}}. \tag{B3}$$

The corollary result for a procedure that uses a response requirement (concurrent VI VR) is:

$$p = \frac{(\mathrm{VI} + b_1)\mathrm{VR}b_2 + I(\mathrm{VR}b_2 - \mathrm{VI} - b_2)}{\mathrm{VI}(\mathrm{VI} + b_1) + \mathrm{VR}b_1 b_2}, \tag{B4}$$

where b_1 is the average local interresponse time at the VI and b_2 is the average local interresponse time at the VR, assuming a Poisson pattern of responding.

For the schedule values and local response and changeover rates in Herrnstein and Heyman's (1979) study of concurrent VI VR, Equation B4 typically predicts values of p greater than .50. The prediction was confirmed by the results. For the schedule values and changeover rates in the present experiment, Equation B3 accounts for only about 70% of the variance in the observed choice proportions, but this equation presupposes *normative* matching—that is, no bias or undermatching or overmatching.

Equation B3 predicts exclusive preference for the VT when the size of its time requirement equals that of the VRT. Correspondingly, Equation B4 predicts exclusive preference for the VI when VI + b_1 equals VRb_2—that is to say, when the reinforcement rates are equal for the two alternatives. Myerson and Miezin (1980) tested this prediction against the results from our earlier report. For the condition that most closely approximated equal reinforcement rates, the subject chose the interval schedule exclusively, as predicted.