20 IS TIME ALLOCATION UNCONDITIONED BEHAVIOR?

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In experiments that provide two reinforcement schedules (concurrent schedules), subjects switch back and forth between the two alternatives, responding for a while at each. The result that has received most attention is that the overall division of behavior between the two schedules approximates the overall division of reinforcements between the two schedules (Herrnstein, 1961, 1970). That is, $B_1/B_2 = R_1/R_2$, where B_i is either total responses or total time at schedule i, and R_i is total reinforcement at schedule i. Called *matching*, this result is often referred to as a law because of its empirical generality. For example, the available evidence shows that the matching law describes concurrent schedule performance independently of both individual and species differences.

Because of the generality of the matching law, recent theoretical accounts of instrumental behavior invariably include a derivation of the matching predictions (e.g., Killeen, 1982; Myerson and Miezin, 1980; Rachlin, Green, Kagel, and Battalio, 1976; Staddon, 1980). These derivations proceed from quite different assumptions; consequently, current theories of instrumental behavior differ quite markedly. In this chapter three types of theory are assessed according to how well they predict time allocation in concurrent schedules.

Most theories of matching can be placed in one of three groups: (1) theories that focus on the relation between responding and its overall consequences (e.g., Rachlin, 1980; Staddon, 1980); (2) theories that focus on the relation between responding and somewhat less molar consequences, such as the local reinforce-

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ment rates (reinforcement at an alternative divided by time at the alternative, e.g., Herrnstein and Vaughan, 1980; Myerson and Miezin, 1980); and (3) theories that are compatible with the view that time allocation in concurrent schedules in an unconditioned reinforcement effect, that is, the amount of time a subject spends at a schedule is elicited (e.g., Herrnstein, 1979; Killeen, 1982; McDowell and Kessel, 1979). The first two approaches are representative of the emphasis in operant psychology on the law of effect: consequences, however measured, select an adaptive if not optimal pattern of behavior. The third approach differs somewhat from most accounts of instrumental behavior. For example, in Killeen's (in press) derivation of matching, reward elicits a change in activity level (there is no explicit or implicit feedback loop). The predictions for each approach are outlined next.

Matching is derived from overall consequences in theories that assume that reinforced behavior is governed by a maximization principle. For example, Rachlin and his colleagues (Rachlin et al., 1976) have argued that the distribution of behavior that produces the maximum overall good predicts the allocation of time spent responding in a concurrent schedule. In some accounts the overall good is the nominal reinforcement rate (e.g., Rachlin et al., 1976), and in others, the overall good is the overall reinforcement rate plus other factors, such as response cost and leisure (Rachlin, Battalio, Kagel, and Green, 1981). Whether either of these theories is correct can be determined by comparing the reinforcement rate maximizing predictions with performance in schedules that use different contingencies: If subjects maximize reinforcement rate then the predictions and performance should simply converge; alternatively, if subjects maximize some more complex response consequence, for example reinforcement less response cost, then reinforcement rate maximization solutions and performance should systematically covary. For example, if the time allocations that maximized reinforcement rate in three different procedures were shown to be 1:1, 3:1, and 9:1, then time allocation should be least biased in the first procedure and most biased in the third procedure. In effect then the test is an application of Meehl's (1950) method for determining whether a stimulus is reinforcing or not. Also note that the evaluation assumes a dynamic maximizing process: Patterns of behavior that produce larger values of the good to be maximized are more likely to persist than patterns of behavior that produce smaller values. Herrnstein and Vaughan (1980) have aptly labeled this view of the dynamics of matching "literal" maximization theory.

The independent variable in local reinforcement rate theories (Herrnstein and Vaughan, 1980; Myerson and Miezin, 1980; Staddon, 1977) is the discrepancy between the two local reinforcement rates, and the process that leads to matching is equalization of local reinforcement rates. For example, Herrnstein and Vaughan (1980) suggested that subjects switch to the alternative with the higher local reinforcement rate. Over the long run this pattern of switching will produce

equal local reinforcement rates, a result equivalent to matching ($R_1/T_1 = R_2/T_2$ is equivalent to $R_1/R_2 = T_1/T_2$).

Two equalization theory predictions were tested. First, the theory predicts that subjects in concurrent schedules should match even though other distributions of behavior produce higher reinforcement rates. (In contrast, maximization theory predicts that matching will not occur if other distributions produce higher reinforcement rates.) Second, equalization theory predicts that switching should be an orderly function of changes in local reinforcement rates. For example, in concurrent procedures that use variable-interval schedules, local reinforcement rates change as a function of time since the last switch. Therefore, if subjects in experiments are compensating for local reinforcement rate differences, switching probabilities should also change as a function of time. Note that Herrnstein and Vaughan's definition of a literal theory applies here as well: The evaluation of equalization theory assumes that behavior that produces smaller differences between local reinforcement rates prevails over behavior that produces larger differences.

Elicitation models of concurrent schedule performance have not been developed in any detail. It is nevertheless possible to describe conditions that would disprove this approach. First, as with equalization theory, elicitation theory implies that concurrent schedule performance is independent of the overall reinforcement rate. Second, since elicitation is a ballistic as opposed to an adjustive process, switching should not vary dynamically with the consequences of switching. Elicitation theory can therefore be eliminated from consideration if it is shown that time allocation depends on either feedback from the overall consequences of responding or feedback from the consequences of switching.

Mathematical models provide the most practical way to describe the relation between time allocation and its consequences in concurrent schedule experiments. These models, referred to as schedule feedback functions (Baum, 1973), entail two terms. One represents the subject's performance; the other represents the schedule. For example, consider a variable-ratio schedule. The expected reinforcement rate, $E(R)$, is p/VRr, where p is the proportion of time at the schedule and r is the mean interresponse time: The expected reinforcement rate is proportional to the time spent at the schedule. In contrast, interval schedules, which are much more frequently used in concurrent procedures, are mathematically more complex. This is because changeover rate and pattern of responding affect reinforcement rate. The discussion of these issues begins with the derivation of an elementary concurrent schedule model.

INDEPENDENT INTERVAL SCHEDULES

Assume that the experiment uses a concurrent schedule with two independent timers, a continuous-response requirement, and no delay between a changeover

Figure 20-1. The sequence of events in a concurrent VT VT schedule. The rectangular excursions in the first and fourth lines indicate that a timer interval elapsed. The diagonal slashes across the second and third lines indicate that the subject just received a reinforcement. If the subject is present when the interval elapses, a reinforcer is delivered immediately, events *a* and *b*. If the subject is absent when an interval elapses, the reinforcer is held, events *c* and *d*, and not delivered until the subject returns, events *e* and *f*.
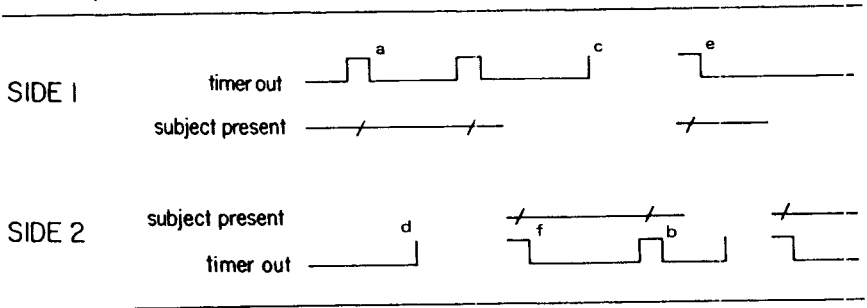

and reinforcement. Figure 20-1 shows the sequential structure of performance in this situation. Sides 1 and 2 refer to the two independently operating variable-interval timers, each distinguished by a stimulus. Lines 2 and 3, labeled "subject present," show that the subject switches back and forth between the two alternatives. This produces two different ways for reinforcement to occur. First, an interval may elapse while the subject is present. This yields an immediate reinforcement, and, simultaneously, the timer restarts with a new interval (events a and b in Figure 20-1). Second, an interval may elapse while the subject is absent (events c and d). In this case the reinforcer is held and the timer is not restarted until the subject returns. Upon returning, the reinforcement is immediately delivered and the timer is restarted (events e and f).

Second, assume an exponential distribution of intervals for each timer, with means V_1 and V_2. This approximates a commonly used schedule (Fleshler and Hoffman, 1962).

Third, assume that the temporal pattern of switching is on the continuum depicted in Figure 20-2. One endpoint is produced by a Poisson process; the other is produced by a subject that switches at precisely time t since the last switch. These two endpoints define two related continuums. First, for the Poisson process, the likelihood of the next switch does not increase as a function of time, which is to say that certainty is minimized. In contrast, for fixed-time switching, the probability of the next switch is 0 at time less than t and 1.0 at t, which is to say that certainty is maximized. Second, reinforcement rate varies with certainty of switching. For a given rate of switching, the Poisson process produces a lower reinforcement rate than any regularly increasing pattern (see Appendix 20A for a discussion of this point), whereas the fixed-time pattern produces a

Figure 20-2. Three different switching patterns in a concurrent schedule. In the first pattern the conditional probability of a switch is constant. This pattern is produced by a Poisson process. In the second pattern the conditional probability of a switch increases. This pattern was produced by a uniform distribution of interchangeover times, with each interval assigned a 10 percent relative frequency. In the third pattern the probability of a switch is 1.0 at 5 seconds. This pattern is produced by an interchangeover time distribution that takes only one value, 5 seconds.

TIME SINCE LAST CHANGEOVER
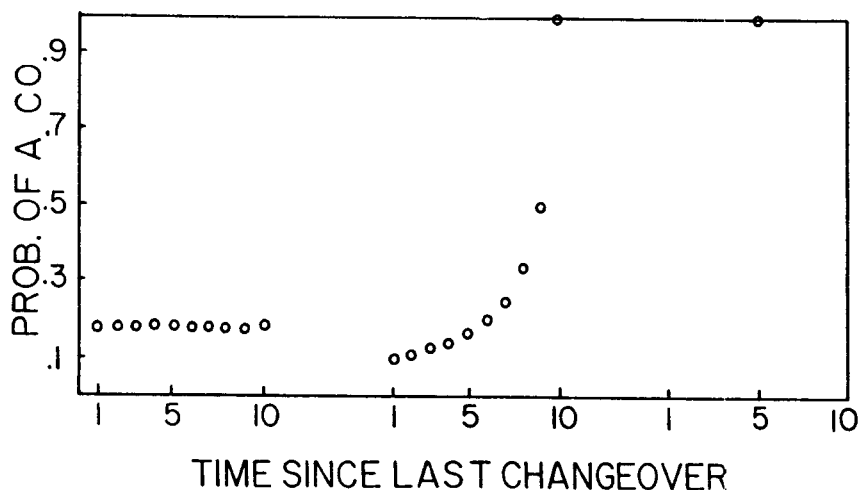
higher reinforcement rate than any other switching pattern (see Appendix 20A). In between these critical values, the certainty of the next switch increases as a function of time, and these patterns produce reinforcement rates that fall along the interval set by Poisson and fixed-time switching. The middle pattern in Figure 20-2 is one example. It is produced by a uniform distribution of interswitch times.

With the foregoing definitions it is possible to derive equations that describe the relation between time allocation and reinforcement rate in concurrent interval schedules for the range of response patterns depicted in Figure 20-2. (Note that this range is likely to include many if not most switching patterns.) The general strategy is to calculate the expected number of reinforcements per visit to an alternative and to distinguish the different ways reinforcement can occur during a visit.

First consider the case when a timer interval elapses while the subject is present. Figure 20-1 shows that these reinforcers are delivered immediately. Therefore their expected number per visit is simply $E(t_i)/V_i$, where $E(t_i)$ is the mean visit time and V_i is the mean timer interval.

Second, a timer can elapse while the subject is not present. This can occur no more than once per absence from the side (since the reinforcer is held), and the expected probability of this event depends on the mean timer interval and the mean rate and pattern of switching back. For Poisson switching, the probability that the timer interval elapsed during an absence is

$$\int_0^\infty \mu_i e^{-\mu_i t} \left(1 - e^{-t/V_j}\right) dt = \frac{1}{1 + \mu_i V_j} , \tag{20.1}$$

where $\mu_i e^{-\mu_i t}$ is the probability that the subject returned at time t, $1 - e^{-t/V_j}$ is the probability that the timer interval elapsed by time t, and $1/V_j$ is the timer rate constant. For the pattern of switching that is most different from Poisson, fixed time, the probability that a timer elapsed during an absence is $1 - e^{-T_i/V_j}$, where T_i is the fixed time the subject spends at the other alternative.

The expected number of reinforcements for the two different ways that reinforcement can occur having now been calculated, the last step is to calculate the expected time per visit. For a Poisson switching pattern this is

$$\int_0^\infty t\mu_i e^{-\mu_i t} dt = \frac{1}{\mu_i} . \tag{20.2}$$

For a fixed-time switching pattern, the expected time per visit is simply T_i.

Therefore the expected reinforcement rate for Poisson switching is

$$\frac{1/V_1\mu_1 + 1/(1 + \mu_2 V_1) + 1/V_2\mu_2 + 1/(1 + \mu_1 V_2)}{1/\mu_1 + 1/\mu_2} , \tag{20.3}$$

where $1/V_i$ is the programmed reinforcement rate at side i, and $1/\mu_i$ is the expected time per visit at side i. For the fixed-time switching pattern, the expected rate is

$$\frac{T_1/V_1 + 1 - e^{-T_2/V_1} + T_2/V_2 + 1 - e^{-T_1/V_2}}{T_1 + T_2} , \tag{20.4}$$

where T_i is the expected time per visit at side i.

Equations (20.3) and (20.4) may be rewritten so that they correspond to the variables usually discussed, the overall allocation of behavior, p, and reinforcement rate. In the notation for Poisson switching, the overall time proportions are $(1/\mu_1)/(1/\mu_1 + 1/\mu_2) = p$ and $(1/\mu_2)/(1/\mu_1 + 1/\mu_2) = 1-p$. While in fixed-time switching notation, these proportions are $T_1/(T_1 + T_2) = p$ and $T_2/(T_1 + T_2) = 1 - p$. Therefore Eq. (20.3) becomes

$$E(R) = \frac{p}{V_1} + \frac{1-p}{V_1 + 1/\mu_2} + \frac{1-p}{V_2} + \frac{p}{V_2 + 1/\mu_1} , \tag{20.3a}$$

and with some algebraic manipulation, Eq. (20.4) is

$$(20.4a)$$

$$E(R) = \frac{p}{V_1} + \frac{1-p}{V_1 + (A_1 - V_1)} + \frac{1-p}{V_2} + \frac{p}{V_2 + (A_2 - V_2)} ,$$

where

$$A_i = T_j / 1 - e^{-T_j/V_i} . \qquad (20.4b)$$

Put this way, the relation between responding and reinforcement rate in concurrent schedules is easily stated in the standard measures.

With probability p the subject is at side 1 when the side 1 timer sets up. This occurs at a mean rate of $1/V_1$. With probability $(1 - p)$ the subject is at side 2 when the timer sets up at side 1. For Poisson switching this introduces a delay equal to the mean time away, $1/\mu_2$, so that these reinforcements occur at rate $1/(V_1 + 1/\mu_2)$. For fixed-time (optimal) switching the delay is

$$T_j / 1 - e^{-T_j/V_i} - V_i . \qquad (20.4c)$$

To get some idea of the effect of different patterns of switching on the delay from setup to reinforcement consider the case when a Poisson subject and fixed-time subject switch at the same rate, that is $\mu_i = 1/T_i$. For typical timer and changeover rates, the fixed time delay is approximately $T_i/2$, whereas the Poisson delay is $1/\mu_i$. This means that the Poisson delay is approximately twice as long for the same rate of switching. However, the effect on overall reinforcement rate (discussed more fully later) is quite small since the corresponding percentage difference between the two interreinforcement intervals is approximately $T_i/2V_j$, and this fraction (usually less than 1/10) only applies to reinforcers that set up at the unattended alternative.

Equations (20.3a) and (20.4a) show that the overall reinforcement rate in concurrent schedules depends on the absolute local switching rates (e.g., $1/T_i$) and on the relative magnitude of the local switching rates. These measures, however, are usually discussed somewhat differently as the overall changeover rate (e.g., $2/(T_1 + T_2)$ in fixed-time notation) and overall time allocation p.

Figures 20-3 and 20-4 show how the expected reinforcement rate changes with the usual measures, overall changeover rate, and overall time allocation in a concurrent interval schedule with a continuous response requirement (conc VT 60 seconds VT 180 seconds). The x axis in each figure is time allocation, and the y axis is expected reinforcement rate. The label I identifies the sum of the local changeover rates, and for each curve this sum is constant. This quantity has been discussed by a number of investigators (e.g., Heyman, 1977, 1979; Hunter and Davison, 1978; Myerson and Miezin, 1980; Nevin and Baum, 1980), and it seems that in situations in which the overall programmed reinforcement rates are con-

Figure 20-3. The expected reinforcement rate in a concurrent VT 60-second VT 180-second schedule as a function of time spent at the VT 60-second schedule. The switching pattern is Poisson. Each curve corresponds to a different level of the tendency to switch, I. For example, at I = 10 second, the local rates of switching sum to 6/minute (and the overall rate is $2p(1 - p)/I$, see text).

EXPECTED REINFORCEMENT RATE
ON A CONCURRENT VT 60 sec VT 180 sec SCHEDULE


PROPORTION OF TIME ON VT 60 sec SCHEDULE
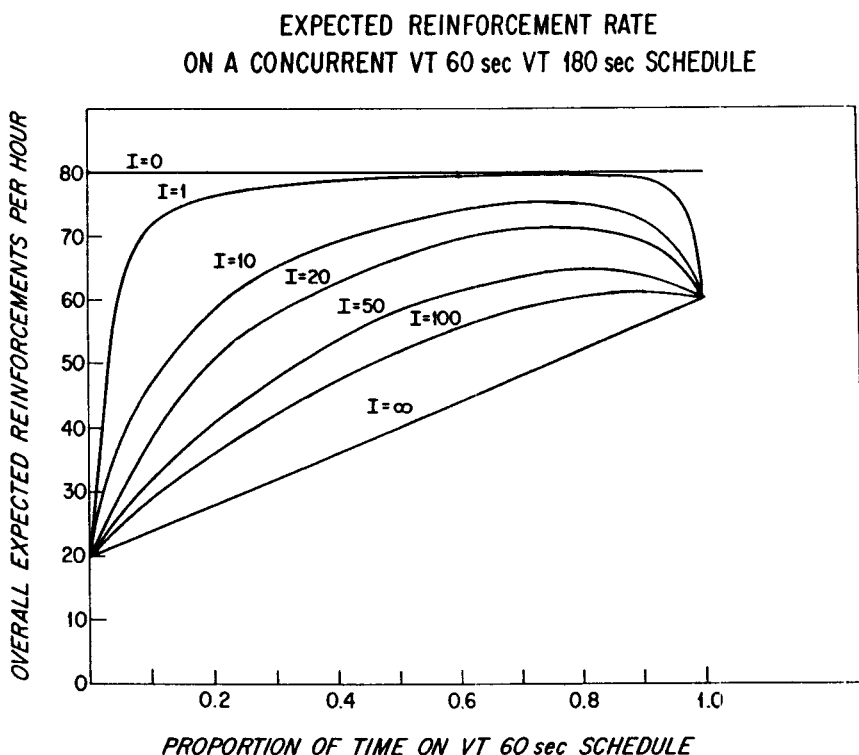
stant, local changeover rates vary reciprocally so that their sum $(1/T_1 + 1/T_2$ or $\mu_1 + \mu_2)$ is approximately constant. This implies (see Appendix 20B for proof) that overall changeover rate is a function of overall time allocation according to the approximation

$$x = 2/(E(t_1) + E(t_2)) \cong 2p(1 - p)/I \quad . \tag{20.5}$$

where x is overall changeover rate, I is equal to the reciprocal of the approximately constant sum of the two local changeover rates. That is, $I = 1/(\mu_1 + \mu_2)$ in the notation for Poisson switching and $I = T_1 T_2/(T_1 + T_2)$ in the notation for fixed-time switching.

Figure 20-4. Expected reinforcement rate in a concurrent VT 60-second VT 180-second schedule with a fixed-time switching process. This pattern of switching is quite different than Poisson switching (see Figure 20-2), yet the consequences of the two patterns are virtually the same. This finding suggests that it is unlikely that subjects learn the optimal switching pattern in concurrent schedules.
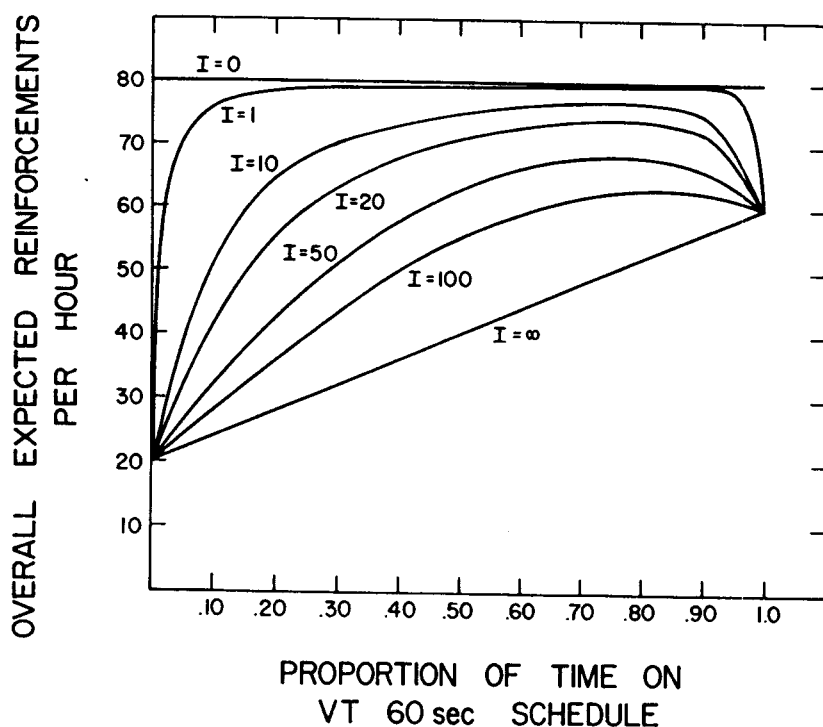
Figures 20-3 and 20-4 show (1) that for both patterns of switching, large changes in time allocation produce small changes in overall reinforcement rate; (2) different patterns of switching produce nearly the same reinforcement rate; and (3) relatedly, the relation between time allocation and overall reinforcement rate is independent of the pattern of switching. Moreover, the effect of time allocation on reinforcement is at a minimum at changeover rates that are representative of experimental subjects. For example, in a study by Brownstein and Pliskoff (1968), the median changeover rates corresponded to a value of I of about 6.6 seconds, and in a VT 60-second VT 180-second condition, the subjects, pigeons, shifted from about a 0.50 to 0.75 division of time. This means that nearly perfect matching was accompanied by no more than a 2 percent change in overall reinforcement rate. Similarly, given the schedule values and changeover

rates, optimal and purely random (Poisson) switching could not have produced more than a 2 percent difference in reinforcement rates.

Figures 20-3 and 20-4, then, do not lend much support to theories of concurrent schedule performance that include overall reinforcement rate as an independent variable. However, these figures do establish a methodological point. Rachlin (1979) questioned whether the equation given here as (20.3a) provided a general description of concurrent interval schedule performance since it might be the case that different patterns of switching yield different reinforcement rate feedback functions. Comparison of Eq. (20.3a) and Eq. (20.4a) settles the issue. If the optimal switching pattern (fixed time) and the contingency independent pattern (Poisson) produce nearly identical expected reinforcement rates, it follows that the generality of the equations does not depend on the temporal pattern of switching. Put somewhat differently: Since pattern of switching has virtually no effect on reinforcement rate, both the Poisson and fixed-time models will accurately predict the relation between time allocation and overall reinforcement rate.

Equations (20.3a) and (20.4a) represent an elementary concurrent variable-interval schedule. In experiments there are typically additional contingencies. The most frequent additions are a discrete response requirement, such as a lever press, and a changeover penalty, called the changeover delay. The response requirement does not appear to affect the allocation of behavior (see, e.g., Brownstein and Pliskoff, 1968); the changeover delay, however, does.

The changeover delay is a changeover-initiated period, usually quite brief, during which reinforcement is withheld. This contingency greatly reduces switching rates and under certain circumstances can have a pronounced effect on how subjects allocate their behavior. The typical result is that if the subject divides its time more or less equally between the alternatives, adding a changeover delay or increasing its duration will differentiate the amounts of time the subject spends responding so that time allocation will match reinforcement allocation. Consequently, the changeover-delay duration has frequently figured in discussions of deviations from matching (e.g., Baum, 1974, 1979; de Villiers, 1977), and some authors have suggested that this contingency is a necessary condition for matching (e.g., Shimp, 1975). However, it is also the case that subjects will match in procedures that do not use the changeover delay (e.g., Bradshaw, Szabadi, and Bevan, 1976; Findley, 1958; Heyman, 1979). Possibly, then, the changeover delay amplifies the consequences of time allocation (Rachlin et al., 1976, originally suggested this) so that for some subjects, the less sensitive, the changeover delay is a necessary condition for matching, whereas for other subjects, the more acute, the changeover delay is not a necessary condition. This hypothesis can be evaluated by calculating the effects of the changeover delay on expected reinforcement rate.

For the fixed-time switching pattern it is possible to determine the effects of the changeover delay on reinforcement rate. In addition, this pattern may not be

greatly unlike actual patterns of switching, because subjects learn not to switch during the delay period (Baum and Rachlin, 1969). The derivation closely follows the derivation of Eq. (20.4) and is also similar to Houston and McNamara's account (1981).

As in the previous models, it is necessary to distinguish the different ways for reinforcement to occur. First, some reinforcers are not postponed by the changeover delay. This is possible if the subject is present and the changeover delay has elapsed. The expected number of times this will happen per visit at alternative i is $(T_i - C_i)/V_i$, where C_i is the duration of the changeover-delay requirement.

Second, some reinforcers are postponed by the changeover delay. This occurs either when the reinforcement timer elapses while the subject is at the other alternative or it occurs just following a changeover during the delay period. These two possibilities are restricted to the interval $T_j + C_i$, where C_i is the delay duration at one alternative and T_j is the fixed time the subject spends at the other alternative. Therefore the probability that the reinforcer is postponed by the changeover contingency is $1 - e^{-(T_j + C_i)/V_i}$. This exhausts the different ways for reinforcement to occur. The expected reinforcement rate is, then,

$$E(R) = \frac{T_1 - C_1}{V_1(T_1 + T_2)} + \frac{1 - e^{-(T_2 + C_1)/V_1}}{T_1 + T_2} + \frac{T_2 - C_2}{V_2(T_1 + T_2)} +$$

$$\frac{1 - e^{-(T_1 + C_2)/V_2}}{T_1 + T_2} . \tag{20.6}$$

Although Eq. (20.6) is superior to Eqs. (20.3a) and (20.4a) in that it takes into account the changeover delay, it is an incomplete description of performance. Missing is the correlation between the duration of the changeover-delay requirement and the changeover rate: An increase in delay produces a decrease in rate. A number of investigators have quantified this relation (see Hunter and Davison, 1978; Stubbs, Pliskoff, and Reid, 1977); however, Eq. (20.5) gives a somewhat simpler account.

Recall that according to Eq. (20.5), changeover rate varies with time allocation and the reciprocal of the constant sum of the changeover rates, called I for inertia. The magnitude of this constant will depend, in part, on procedural factors, such as the distance between the alternatives, the delay requirement, and so on. The simplest possible structure is $I = a + b(C)$. That is, the tendency to switch, I, varies linearly with the delay requirement:

$$\frac{2}{E(T_1) + E(T_2)} = \frac{2p(1-p)}{I} = \frac{2p(1-p)}{a + b(C)} , \tag{20.7}$$

where $E(T_i)$ is the expected time per visit to side i.

Figure 20-5. The relation between changeover delay (COD) duration and changeover rate. On the x axis is the changeover requirement; on the y axis is the changeover rate in reciprocal form (cycle duration). Rat s-1 is from an experiment conducted by Shull and Pliskoff (1967); Pigeon 10838 is from an experiment conducted by Silberberg and Schrot (1974).
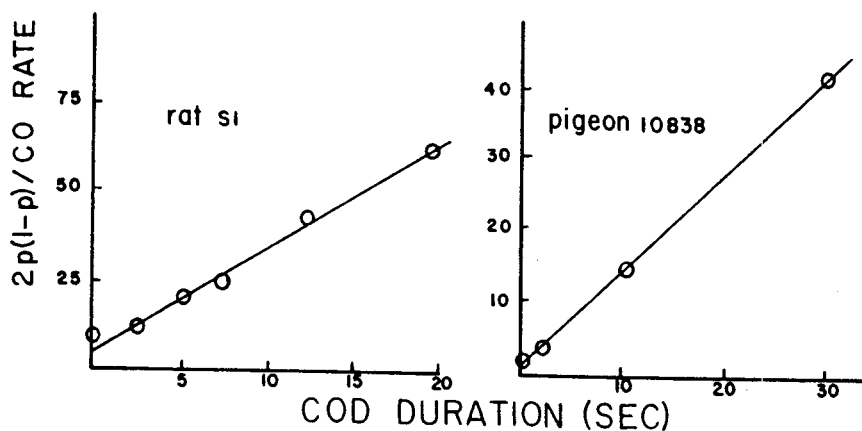

Figure (20-5) shows the fit of Eq. (20.7) to some representative data. On the x axis is the duration of the changeover delay requirement. On the y axis changeover rate is plotted in reciprocal form (the sum of the two average visit times, sometimes called cycle time). Rat s-1 is from a study by Shull and Pliskoff (1967) in which the changeover delay was varied from 0.0 to 20 seconds. Pigeon 10838 is from a study by Silberberg and Schrot (1974) in which the changeover delay was varied from 0.0 to 30 seconds. Equation (20.7) predicts changeover rate quite precisely, except for a slight tendency to underestimate the rate in the 0.0-seconds changeover-delay condition (that is, no changeover-delay requirement). Note, that Eq. (20.7) assumes a Markov model of switching (see Heyman, 1979). In contrast, momentary maximizing theories and melioration theories of switching suggest (if not imply) a feedback driven, adjustive switching process.

Figure 20-6 shows the relation between time allocation, changeover delay, and overall reinforcement rate for a conc VT 60-second VT 180-second schedule. The y axis is the expected overall reinforcement rate. The changeover delay was varied from 0 to 27 seconds, and changeover rate was determined from the best-fitting parameter values for I in a continuous response procedure in which the changeover delay was varied (Brownstein and Pliskoff, 1968).

Figure 20-6 shows that for typical changeover delays, the region circumscribed by the first and third curve, changes in time allocation have little effect on the overall reinforcement rate. For example, at a 3-second changeover delay, a shift from 0.60 to 0.75 in time spent at the VT 60-second alternative increases

Figure 20-6. The relation between time allocation and reinforcement rate in a concurrent VT 60-second VT 180-second schedule that uses a changeover delay. The changeover delay does little to change the overall consequences of time allocation, yet it frequently has a large effect on time allocation.
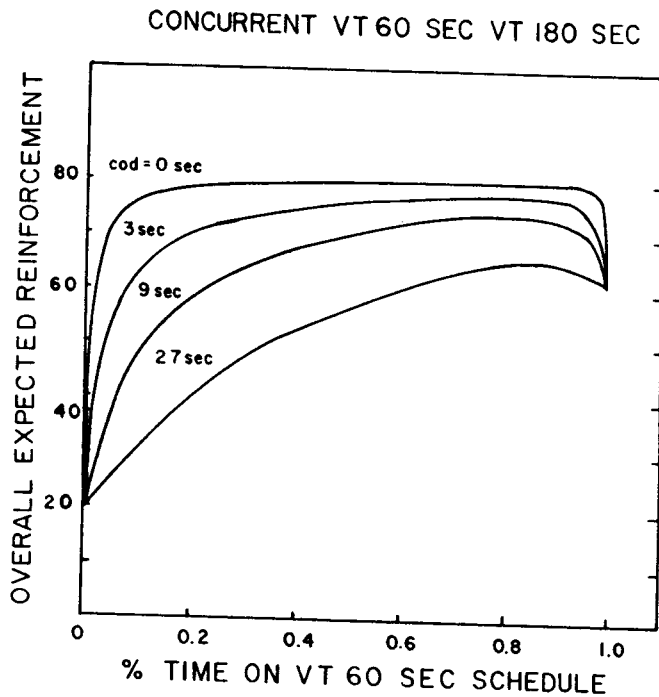

CONCURRENT VT 60 SEC VT 180 SEC

Table 20-1. Overall expected reinforcement rate (reinforcements/hour) in a concurrent VT 60-second VT 180-second schedule as a function of time allocation p and changeover delay duration. The results were generated by Eq. (20.6). The visit times were set to I/p and $I/(1-p)$. I was determined by Eq. (20.7), with the parameters $a = 1.0$ and $b = 1.1$.

| COD Duration | Time on VT 60-Second Schedule and Overall Reinforcements per Hour | | | | |
	.50	.60	.70	.75	.80
0	77.71	78.27	78.56	78.61	78.56
3	75.90	76.20	77.27	77.46	77.57
9	69.62	71.40	72.98	73.40	73.74
27	56.60	60.10	61.66	63.07	64.80

the expected reinforcement rate by less than 2 percent. Table 20-1 provides some additional numerical values.

CONCURRENT SCHEDULE PERFORMANCE, MATCHING AND MAXIMIZING PREDICTIONS

The figures in the preceding section show how changes in the allocation of time responding affect overall reinforcement rate. In this section the allocation of time that maximizes the overall reinforcement rate is calculated. The calculations are then compared with data from three procedures, which differ in terms of the contingency between time allocation and overall reinforcement rate. That is, each procedure has a different maximizing solution. If subjects respond so as to maximize overall reinforcement rate then the predictions and observations should converge. Similarly, if subjects maximize some more complex quantity, such as reinforcement rate minus the costs of responding, then predictions and observations should vary systematically. Of course, the overall consequences of responding, however measured, need not exert any control on time allocation. For example, the acquisition models of Herrnstein (1979), Killeen (1982) and McDowell and Kessel (1979) are blind to the differences that distinguish the three procedures analyzed next, and, consequently, these models predict that subjects should come to the same asymptotic time allocations in three quite different contingencies.

The relation between time allocation and overall reinforcement rate maximization was evaluated in the following way. First, the division of time that maximizes reinforcement rate was determined. This was done by varying p in the schedule reinforcement rate feedback equations. Second, for the value of p that maximized reinforcement rate, the corresponding reinforcement rates for alternative 1 and alternative 2 were extracted. Third, the maximizing solutions were plotted as a function of their corresponding reinforcement rates according to the linear model (Baum, 1974):

$$\log T_1 / T_2 \text{ max} = \log a + b \log R_1 / R_2 \text{ max} , \qquad (20.8)$$

where T_i is the optimal amount of time spent at alternative i and R_1 plus R_2 sum to the maximum reinforcement rate. Fourth, the best-fitting parameter values (Eq. (20.8)) for the maximizing solutions were compared to the best-fitting parameter values (Eq. (20.8)) for the typical subject. In other words, Eq. (20.8) measured whether subjects in experiments maximized overall reinforcement rate.

The three different procedures that are analyzed in this way are concurrent variable-interval with two independent timers, concurrent variable-interval with interdependent timers, and a concurrent procedure in which one alternative is a variable-interval schedule and the other is a variable-ratio schedule (conc VI VR). In addition, for the independent-interval procedure, there is an analysis of

the effects of the changeover delay and response requirement on the maximizing solutions.

Maximizing in Concurrent VI VI

Figures 20–7 and 20–8 show the maximizing policies for experiments with a continuous-response requirement and no changeover delay. Figure 20–7 is for fixed-time switching (Eq. (20.4a)), and Figure 20–8 is for Poisson switching (EQ. (20.3a)). The schedule values, V_1 and V_2, were varied so as to maintain a constant programmed reinforcement rate of 80/hour, and I was set at two different values. The smaller one, 4.3 seconds, corresponds to the parameters for

Figure 20–7. A comparison of typical performance and reinforcement rate maximization predictions in concurrent VT VT schedules. The maximization predictions were generated by Eq. (20.4a) for fixed-time switching. The description of the typical subject is based on Baum's (1979) review. The shallower slope corresponds to a lower changeover rate.
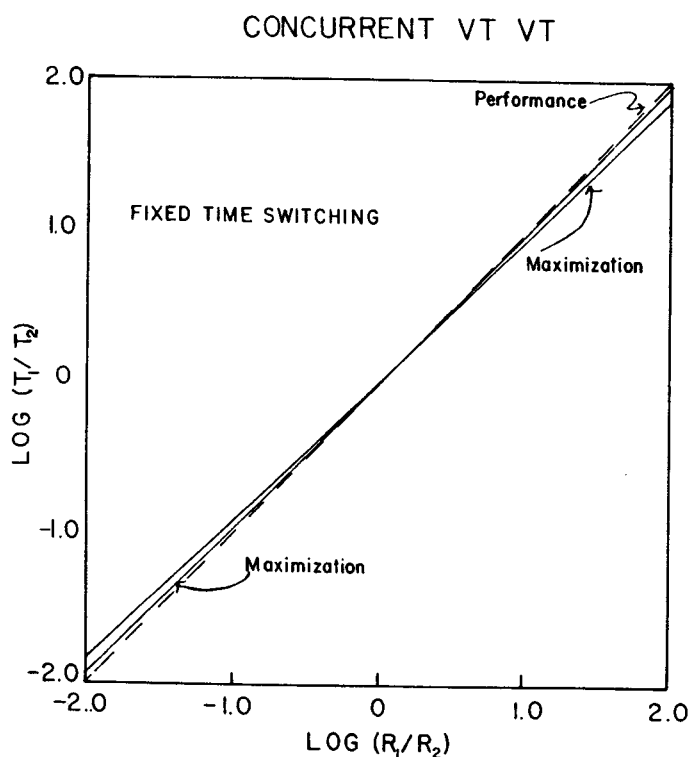

CONCURRENT VT VT

Figure 20-8. A comparison of typical performance and reinforcement rate maximization predictions (Poisson switching) in concurrent VT VT schedules. The maximization predictions were generated by Eq. (20.3a) (Poisson switching). Note that the maximization solutions are virtually independent of pattern of switching. Put somewhat differently, large differences in response pattern in concurrent schedules yield only small differences in outcome.
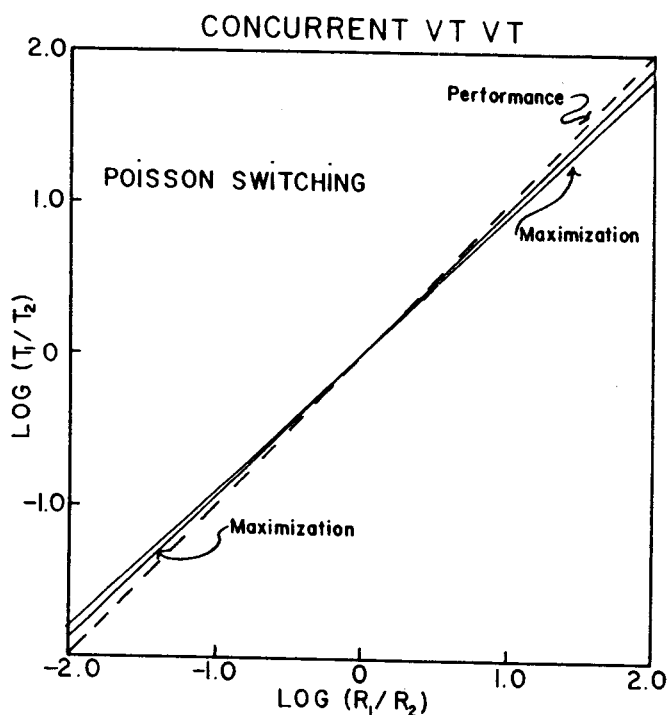
I in an experiment with pigeons (Brownstein and Pliskoff, 1968), and it sets changeover rate at approximately 7/minute for $p = 1/2$. The larger value, $I = 34$ seconds, gives a changeover rate of about 1/minute at $p = 1/2$. The continuous diagonals are the best-fitting straight lines to the ratios of time spent responding that maximized reinforcement rate, and the broken diagonals are the best-fitting straight lines to the ratios of time spent responding for the typical subject.

The parameters for the typical subject are based on reviews by Baum (1974, 1979) and de Villiers (1977). According to those authors, fitting Eq. (20.8) to the available time-allocation data yields a median slope of approximately 1.0 and a median intercept, for experiments with symmetrical response requirements, of approximately 0.0. In other words, time allocation in concurrent schedules does not systematically vary from simple matching (slope of 1.0 and intercept of 0.0).

Equation (20.8) reveals that the maximizing solutions closely approximated performance in concurrent interval schedules. However, the linear model also reveals that the discrepancies, albeit small, were systematic. First, the slopes for the maximizing solutions fell slightly short of 1.0, with lower slopes for lower changeover rates. For fixed-time switching, the slopes were 0.93 and 0.98; for Poisson switching, the slopes were 0.91 and 0.96. Second, the relation between $\log T_1/T_2$ max and $\log R_1/R_2$ max was not precisely linear. Rather, there was the suggestion of a sigmoidal function. In contrast, the empirical deviations from Equation (20.8) are symmetrical (see Baum, 1979).

Figure 20-9 shows the maximizing predictions for procedures that use a changeover delay. As pointed out previously, the changeover delay has the effect of shifting time allocation from indifference, $1/1$, to matching, R_1/R_2, so that from the point of view of maximization theory, the changeover delay should have a sizable effect on the maximizing solutions. Comparison of Figure 20-9 with Figures 20-7 and 20-8, however, show that the changeover delay has a rather modest effect of the relation between time allocation and reinforcement allocation. For example, without the changeover delay, the slopes range from about 0.91 to 0.98, and with the changeover delay, the slopes range from about 1.1 to 1.2.

Figure 20-10 displays the maximizing solutions for concurrent schedules that have a changeover delay and a response requirement. In this type of procedure each alternative has a manipulandum and the subject switches back and forth between them. Reinforcement occurs either after a changeover or after an inter-response time on one of the manipulandums. Formally, the analysis of inter-response times is the same as the previous analysis of interchangeover times (Eqs. (20.3), (20.4) and (20.6)). For example, an exponential interresponse time distribution extends the expected interreinforcement interval by the mean inter-response time (see Herrnstein and Heyman, 1979; Nevin and Baum, 1980).

Figure 20-10 shows that the slopes fit to the maximizing solutions vary as a function of local response rate. At high local response rates (120/minute), the slope is greater than 1.0, and at low local response rates (20/minute) the slope is lower than 1.0. Since the lines fit to the maximizing solutions cross in this way, there may exist some combination of local response rates and changeover values for which matching and maximizing are identical. However, the evidence presented in this chapter indicates that the similarity of matching and maximizing in independent schedules is fortuitous.

Maximizing in Interdependent VI VI

Figures 20-7 to 20-10 show the maximizing solutions for concurrent schedules with independent timers. Next schedules are analyzed in which the alternatives are not independent. The purpose of this modification is to ensure that the subject receives the programmed relative reinforcement rate (see, e.g., Stubbs and Pliskoff, 1969, and subsequent discussion).

Figure 20-9. Performance and maximization theory predictions in procedures that use a changeover delay. The changeover delay can have a quite sizable effect on time allocation, yet it has a small effect on maximization policy. This is consistent with the general finding that large differences in behavior produce virtually the same outcomes in concurrent schedules. Note that increasing the changeover delay duration does not yield steeper slopes because of the way that the delay interacts with obtained reinforcement proportions (see Herrnstein, 1970).
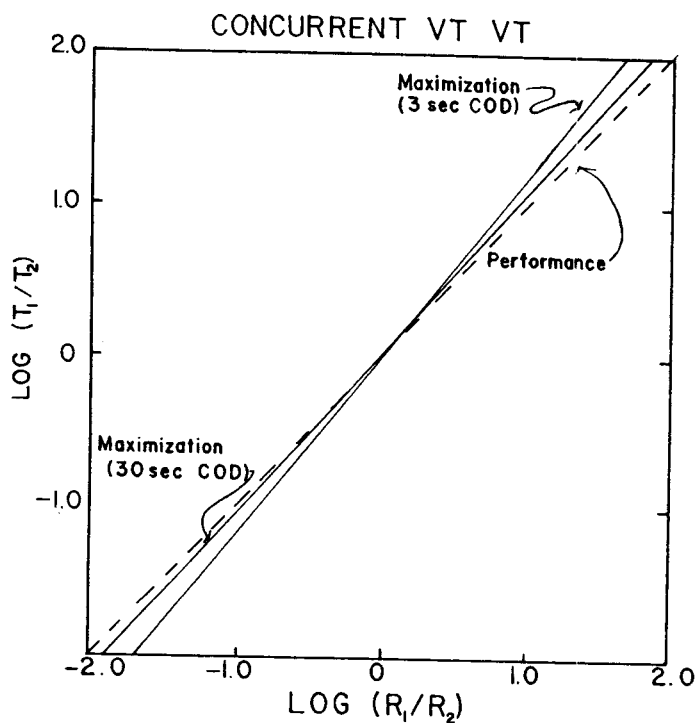
An interdependent procedure can be arranged in two nearly equivalent ways (Fantino, Squires, Delbruck, and Peterson, 1972; Stubbs and Pliskoff, 1969). The model presented (Eq. (20.9)) is based on Fantino et al.'s (1972) method. These researchers used two variable-interval timers. When one timer set up a reinforcer, the other was halted and not restarted until the reinforcer was collected. Thus, if the unattended timer was primed, the attended timer stopped running also. This method ensures that reinforcements are obtained in the order that they are made available, which, in turn, implies that the obtained reinforcement proportions are equal to the arranged reinforcement proportions. The method arranged by Stubbs and Pliskoff (1969) results in a nearly identical con-

Figure 20-10. Performance and maximization theory in concurrent schedules that use a changeover delay and a response requirement. The tendency to switch, I, was calculated according to Eq. (20.7), with the parameters $a = 1.0$ and $b = 1.1$ (Shull and Pliskoff, 1967). The maximization predictions vary with local response rate and for some combinations of changeover delay duration and interresponse time duration, it appears logically possible for performance and maximization to converge.
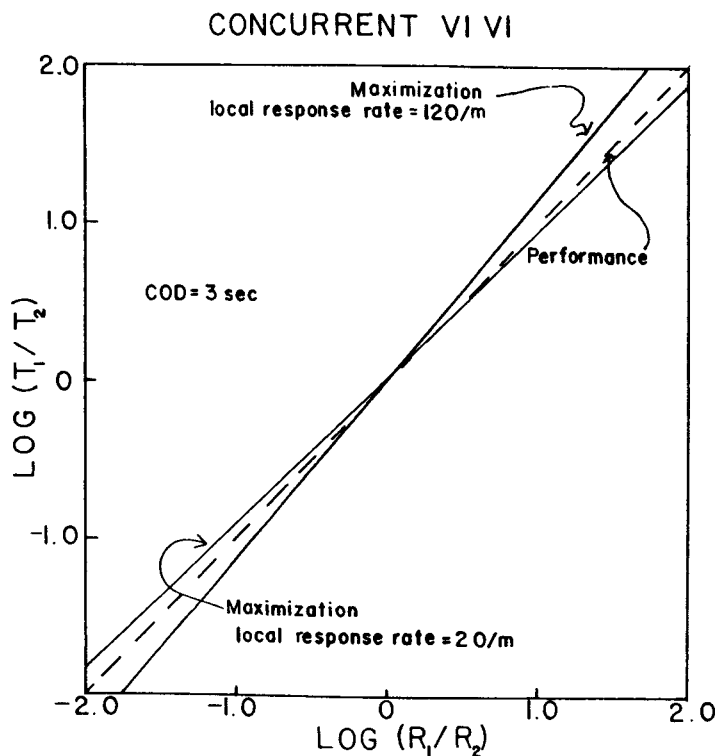
tingency: (1) Reinforcement availability at the unattended alternative prevents reinforcement at the attended alternative; (2) reinforcements are obtained in the arranged order; (3) and obtained reinforcement proportions equal arranged reinforcement proportions. The only difference is that Stubbs and Pliskoff used a single timer so that a reinforcement rather than a changeover resets the expected time to the next reinforcement (see Staddon, Hinson, and Kram, 1981, for a fuller discussion of this point). However, this difference has little effect on the relation between time allocation and reinforcement rate, so that Eq. (20.9) quite accurately predicts the expected reinforcement rate for both methods of arranging interdependent schedules (see, e.g., Heyman and Luce, 1979).

Appendix 20C provides a derivation of the equation for expected reinforcement rate in an interdependent procedure. The result is as follows (also see Heyman and Luce, 1979):

$$E(R) = \frac{p}{V_1} - \frac{pI}{V_1(I+(1-p)V_2)} + \frac{1-p}{V_1 + 1/\mu_2} + \frac{1-p}{V_2}$$

$$- \frac{(1-p)I}{V_2(I+pV_1)} + \frac{p}{V_2 + 1/\mu_1} . \tag{20.9}$$

The second and fifth terms give the expected rate of loss at an attended but not running timer. For example, while the subject is at side 1, the probability that the timer there has come to a halt increases according to the expression given by Eq. (20.1). Once it stops, it remains so until the next switch, and this event has the expectation given by Eq. (20.2). The result is included in Eq. (20.9). Note that this model differs quite markedly from the equations for independent timer procedures. For example, if the subject never switches in an interdependent procedure, reinforcement rate decreases to zero.

Figure 20-11 shows the relation between typical performance and reinforcement rate maximization in interdependent schedules. The maximization solutions reflect the structure of the contingency. That is, since not switching reduces reinforcement rate at both sides, the subject must not spend too much time at the richer alternative. Thus the optimal policy is to allocate time about midway between 1/1 (indifference) and V_1/V_2 (perfect matching). Subjects in experiments do not allocate time according to this contingency, however. Rather, as in independent procedures, time allocation approximates reinforcement allocation. For example, in the most exhaustive review to date, Baum (1979) concluded that time allocation was the same in independent schedules and schedules that maintained a fixed relative reinforcement rate.

Maximizing in Concurrent VI VR

The third procedure in this comparison uses a ratio schedule on one alternative and an interval schedule on the other alternative (conc VI VR). On the ratio schedule, reinforcement rate is proportional to time spent responding so that changes in time allocation produce large changes in reinforcement rate. In contrast, Figures 20-3 and 20-6 showed that on interval schedules changes in time allocation have little effect on reinforcement rate. Accordingly, it is not unreasonable to expect subjects to allocate time rather differently when a ratio schedule controls reinforcement.

The equation for expected reinforcement rate on the ratio schedule is $E(R) = (1-p)/\mathrm{VR}r_2$, where $1-p$ is the proportion of time spent responding, VR is the mean ratio requirement, and r_2 is the mean local interresponse time. Rein-

Figure 20-11. Performance and maximization theory in interdependent interval schedules. The characterization of performance is based on Baum's (1979) review. Maximization theory predictions diverge widely from how subjects actually behave.
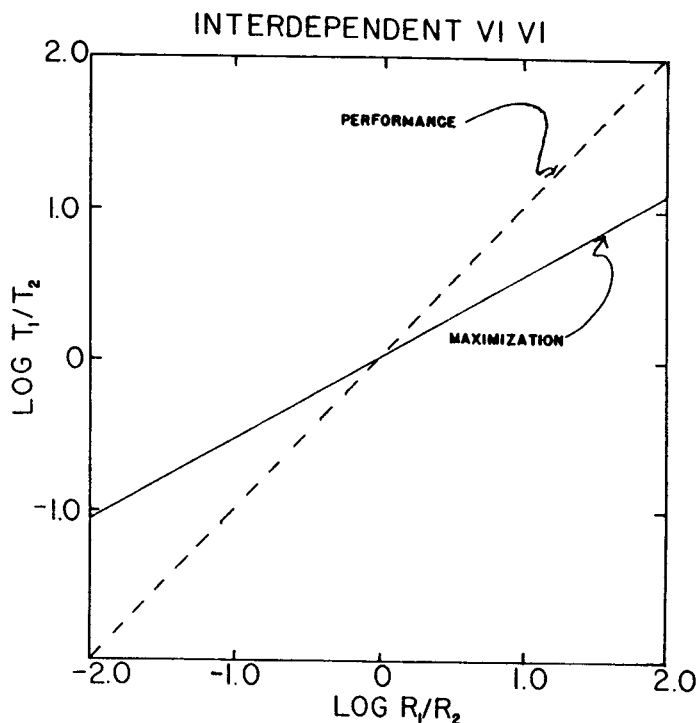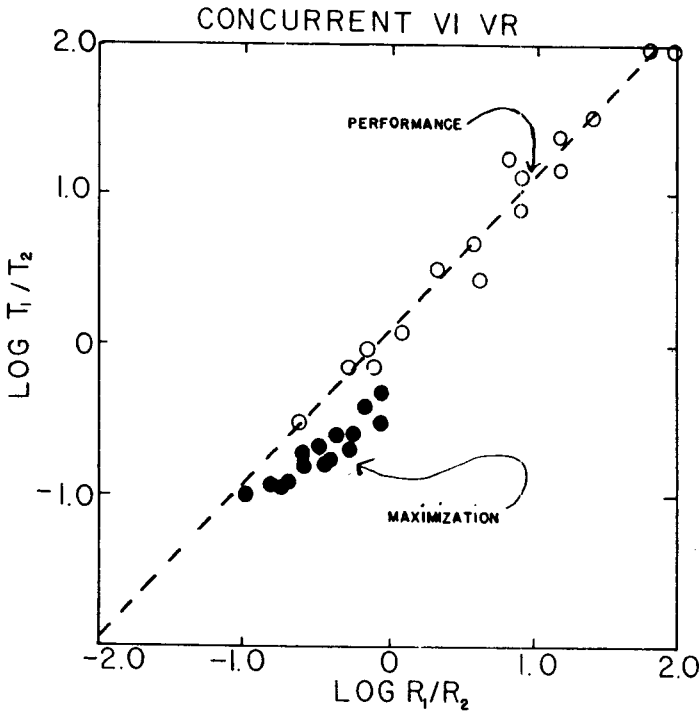

forcement rate on the interval schedule is as previously described in Eqs. (20.3) and (20.4). Therefore, the expected overall reinforcement rate for a conc VI VR is

$$E(R) = p/(VI + r_1) + (1 - p)/(VI + 1/\mu_2) + (1 - p)/VR r_2 \quad , \quad (20.10)$$

where r_1 is the local interresponse time on the VI alternative, r_2 is the local interresponse time on the VR alternative, and the other symbols are as defined previously.

The first two expressions give the reinforcement rate on the VI alternative. With probability p the subject receives rate $1/(VI + r_1)$, the mean programmed interval plus the mean local interresponse time, and with probability $1 - p$, the subject receives rate $1/(VI + 1/\mu_2)$, the mean programmed interval plus the mean time per visit on the ratio alternative. Note that these two rates are not

Figure 20-12. Performance and maximization theory predictions in a concurrent VI VR experiment (Herrnstein and Heyman, 1979). The open symbols show the time-allocation measures for individual subjects. The filled symbols show reinforcement rate maximization predictions.

very different from each other for typical schedule and behavior rates. The third expression is the rate on the ratio alternative. In contrast to the interval schedule, reinforcement rate is zero if the subject is absent, and this occurs with probability p.

Figure 20-12 compares the maximizing solutions with performance in a concurrent VI VR experiment (Herrnstein and Heyman, 1979). The maximizing solutions were derived from Eq. (20.9) (filled symbols) and the subjects were pigeons responding for grain (open symbols). The coordinates are in terms of the ratio VI/VR so that positive values on the y axis indicate that more time was spent on the VI alternative and negative values indicate that more time was spent on the VR alternative.

The maximizing predictions invariably fall into the lower left quadrant of the figure, with a slope of 0.72 for the best-fitting line. This means that in order to maximize overall reinforcement rate, the subjects should have spent most of

their time on the VR alternative and from time to time briefly sampled the VI alternative to check if a reinforcer had set up there.

In contrast to these predictions, the subjects spent most of their time on the VI alternative and did so in a way that was consistent with performance in concurrent-interval schedules. The slope of the best-fitting line is 1.04, which is about the value found in independent and interdependent interval schedules. This means that the allocation of time spent responding did not respect the differences in reinforcement contingencies. Moreover, the costs of contingency indifference were rather great: On the average, the subjects would have received about 60 more reinforcements an hour by maximizing (see Herrnstein and Heyman, 1979).

SUMMARY AND OTHER APPROACHES

Figures 20-7 to 20-12 show that in three quite different procedures subjects responded so that the ratios of time spent at each alternative were equal to the ratios of reinforcement obtained at each alternative; that is, $T_1/T_2 = R_1/R_2$ (correcting, if necessary, for bias). In contrast, there was no systematic relation between the maximizing policies and time allocation. This absence of covariation with the maximizing solutions implies that subjects in concurrent schedules do not maximize value, where value includes the overall reinforcement rate. Put somewhat differently, these results say that in concurrent schedules time allocation is not controlled by its overall consequences.

The conclusion that subjects in operant reinforcement schedules are not controlled by the molar consequences of their behavior is at odds with recent economic (Rachlin et al., 1981) and biologically (Staddon, 1980) oriented theories of instrumental (adaptive) behavior. Some support for these theories comes from Staddon and Motheral's (1978) mathematical model of the concurrent VI VI contingency. Since this model does not agree with the mathematics presented in this chapter, some comments are necessary.

Staddon and Motheral claim that the expected rate of reinforcement in a concurrent schedule with independent timers is

$$E(R) = 1/(V_1 + 1/x) + 1/(V_2 + 1/y) \quad , \qquad (20.11)$$

where x and y are the overall response rates (responses at a side divided by the entire session time, in contrast to the local response rates, discussed in the previous sections of this chapter, which are responses at a side divided by the portion of time spent at a side). Staddon and Motheral's equation says that the expected time between reinforcements is the mean timer interval V_i plus the mean overall interresponse time. But this cannot possibly be correct since in concurrent schedules reinforcement occurs after a local interresponse time or an interchangeover time (there are no other possibilities), and the overall response

rates have no necessary connection with either of these measures. For example, it is quite possible for two subjects to produce equal overall response rates but obtain quite different reinforcement rates, because one switches frequently and the other switches infrequently.

Staddon and Motheral (1979) anticipated this criticism and write that Eq. (20.11) implies a changeover rate and local response rate. They write that changeover rate is

$$C = 2xy/(x+y) \qquad\qquad (20.11a)$$

and that local response rate is

$$2xy/C \quad , \qquad\qquad (20.11b)$$

where C is the overall changeover rate (Eq. (20.11a)). These definitions are incorrect, however. In other words, according to Staddon and Motheral changeover rate is a function of response rate. However, one of the basic features of concurrent schedule performance is that switching rates and response rates are independent measures. For example, switching rate varies with time allocation, whereas response rates remain approximately constant with changes in time allocation (e.g., Stubbs and Pliskoff, 1969).

EQUALIZATION AND MOMENTARY
MAXIMIZING THEORIES

Equalization theories of matching (e.g., Herrnstein and Vaughan, 1980; Myerson and Miezin, 1980; Staddon, 1977) predict that concurrent schedule performance is independent of molar consequences and that switching will vary in an orderly way with local reinforcement rates. Figures 20–7 to 20–12 confirmed independence. Next the predictions for switching are tested.

In a concurrent schedule that uses interval schedules, the local rates of reinforcement rise and fall with switching. A switch produces an increase, and, then, there is a decline until the next switch. This means that the longer the subject stays at one side, the relatively greater the local rate of reinforcement at the other side. Therefore, if equalization theory describes concurrent schedule performance, the probability of a changeover should increase as a function of time since the last changeover. Figure 20–13 tests whether this prediction holds.

The figure shows the temporal pattern of switching in three different concurrent-schedule experiments. On the x axis is the number of responses or amount of time (pigeon B 64) since the last switch; the filled circles are the conditional probabilities of switching from the richer alternative; and the open circles are the conditional probabilities of switching from the leaner schedule.

Pigeons 209 and 241 were in an experiment that used an interdependent schedule (Heyman, 1979), and the temporal pattern for pigeon 209 is representative of the main findings of the study. The switching probabilities were station-

Figure 20-13. Switching in three different concurrent schedule experiments. The x axis is the number of responses since the last changeover or time since the last changeover (pigeon B 64). The y axis is the conditional probability of a switch. The concurrent schedule reinforcement contingency requires that the probability of reinforcement and local rate of reinforcement increase at the unattended side. The figure shows that subjects did not switch in accordance with these consequences.
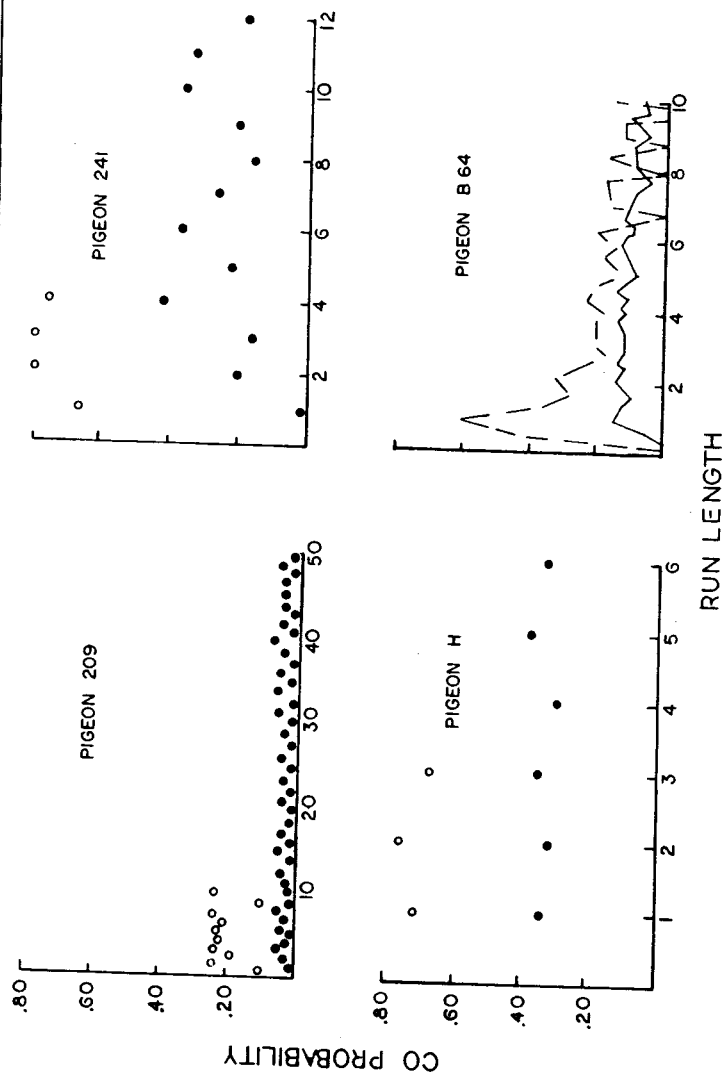
ary (chi-square test) and independent of run length. This means that switching did not depend on changes in local reinforcement rates. The data from pigeon 241 are characteristic of the minority of instances in which switching probabilities were not stationary (3 of 24 cases). The probabilities rose and fell in an alternating pattern. This corresponds with previous reports (e.g., Blough, 1966) of response bursts, but it is incompatible with the changes in local reinforcement rate described in equalization theory.

Pigeon H was in a discrete-trials experiment (Herrnstein, unpublished, described in de Villiers, 1977). Reinforcement was arranged by a concurrent VI 135-second VI 270-second schedule (independent), the intertrial intervals were 22 seconds, and the relative probability of reinforcement for a switch, counting from the last changeover, rose from about 0.50 to about 0.90. Nevertheless, the subjects were no more likely to switch after 6 trials than after 1 trial.

Pigeons B 64, like pigeons 209 and 241, was in an experiment that used an interdependent procedure (Silberberg, Hamilton, Casey, and Ziriax, 1978). The x axis is time, not responses; the broken line plots the probabilities of switching from the leaner schedule; and the solid line plots the conditional probabilities for the richer schedule. Again, the pattern predicted by local reinforcement rate control does not emerge. Instead, switching was either independent of time (from the richer schedule) or showed a peak and then a gradual decline with time (from the poorer schedule).

Figure 20-13 also evaluates a somewhat similar theory of matching, momentary maximizing. In this account (Shimp, 1969), the local reinforcement probabilities that accompany switching are the independent variables, and the psychological principle is that subjects switch to the schedule that is most likely to provide the next reinforcer. For example, in a concurrent variable-time schedule (no response requirement, e.g., Baum and Rachlin, 1969; Brownstein and Pliskoff, 1968) the subject should switch when the probability of reinforcement at the unattended alternative, $1 - e^{-t_i / V_j}$, is greater than the probability of reinforcement at the attended alternative, $1/V_i$, where t is time since the last changeover. For other procedures the calculations are slightly different, but for any concurrent schedule, momentary maximizing predicts that the subject should switch at some target (optimal) time. Assuming a normally distributed error about the target time, it can be shown that the resulting distribution of interchangeover times will yield a regularly increasing pattern when plotted in the coordinates of Figure 20-13 (see Thomas, 1971, for a proof). Figure 20-13, however, shows that in three different concurrent-schedule experiments, the momentary maximizing pattern does not occur.

The data presented in Figure 20-13 indicate that switching is independent of local reinforcement rates and local reinforcement contingencies. In contrast, Shimp (1969) and more recently Silberberg et al. (1978) claim that momentary maximizing theory predicts switching. These authors, however, do not plot their data in terms of the relevant reinforcement probabilities. Rather, they use a cumulative-run length measure that bears no obvious relation with the rein-

forcement contingencies. And, in fact, when the data are replotted in a way that corresponds to how reinforcement is obtained, the momentary maximizing predictions are absent, as in Figure 20-13 (see Nevin, 1979).

CONCLUSION

The basic tenet of operant psychology is that behavior is controlled by its consequences. Accordingly theories of matching have typically assumed that some sort of behavioral consequence determines time allocation in concurrent schedules. Figures 20-7 to 20-12, however, showed that time allocation is independent of its overall consequences, and Figure 20-13 showed that the dynamics of time allocation, switching, is independent of its immediate consequences. Together, these data suggest the conclusions that how much time an organism spends at a reinforcement schedule is an unconditioned effect and that the elicitation processes conform to the matching equation. Finally, since the matching relation was initially formulated in terms of overall response rates (Herrnstein, 1961) something needs to be said about this way of measuring behavioral allocation.

Overall response rate depends on the amount of time spent at the schedule, and it also depends on the topography of the emitted response. For example, a subject in a concurrent schedule experiment may spend exactly the same amount of time at each alternative but respond at very different overall rates because at one side the lever takes 1 second to depress and at the other side the lever takes 5 seconds to depress. In turn, these two dimensions of response rate depend on reinforcement contingencies. Time spent responding, as described previously, is a function of variables having to do with shear reinforcement frequencies, independent of the feedback relation between a response and reinforcement. In contrast, response topography, as is shown next, appears to depend on the feedback relation independently of reinforcement frequency. This is a very different account of response rate than given by cost/benefit analyses (e.g., Baum, 1973; Rachlin, 1980), and this paper concludes with an outline of some of the relevant evidence.

In experiments in which variable-interval and variable-ratio schedules maintain equal reinforcement rates, subjects respond approximately twice as fast for the variable-ratio reinforcement (Herrnstein and Heyman, 1979). Importantly, the local response rates ratio was invariant; it did not depend on absolute reinforcement rates or absolute response rates. In contrast, variable-interval and variable-ratio reinforcers maintained virtually identical amounts of time spent responding (see Figure 20-12 and corresponding discussion). That is, time allocation depended only on the frequencies of reinforcement. These facts are compatible with the view that a reinforcement schedule simultaneously elicits and shapes behavior: Reward impels activity, independent of the contingency, and the contingency shapes activity into an effective response, independent of activity level.

APPENDIX 20A

The pattern of switching affects the probability of reinforcement for a switch but has no affect on reinforcement obtained between switches. Therefore the pattern that produces the highest return on switches will produce the highest overall reinforcement rate. Equations (20A.1)–(20A.3) give the probabilities for the patterns shown in Figure 20-2. For an exponential distribution of inter-changeover times, the expected probability of reinforcement for a switch is

$$1/(1 + V_j \mu_i) \quad . \tag{20A.1}$$

For a uniform distribution of interchangeover times, the expected probability of reinforcement for a switch is

$$\frac{1 - e^{-T_1/V_j} + 1 - e^{-T_2/V_j} \dots + 1 - e^{-T_n/V_j}}{n} \quad , \tag{20A.2}$$

where the average time between switches is $(T_n/2) + 1/2$ and $(T_n/2) + (1/2) = 1/\mu_i$ (Eq. (20A.1)) = T_i (Eq. (20A.3)). For a point-mass distribution of inter-changeover times (fixed-time switching) the expected probability of reinforcement for a switch is equal to:

$$1 - e^{-T_i/V_j} \quad . \tag{20A.3}$$

The relative magnitude of Eqs. (20A.1) – (20A.3) take the order (20A. 3) > (20A.2) > (20A.1). Therefore Poisson switching produces the lowest reinforcement rate and fixed-time switching produces the highest reinforcement rate. In addition Houston and McNamara (1981) have shown that fixed-time switching is the optimal pattern in that no other pattern of switching can produce a higher reinforcement rate. In contrast, Poisson switching produces a lower reinforcement rate than any regularly increasing pattern.

APPENDIX 20B

Overall changeover rate may be written $2/(E(t_1) + E(t_2))$, where $E(t_i)$ is the expected time per visit to side i. Note that the sum of the two local changeover rates is approximately constant (Heyman, 1979; Hunter and Davison, 1978; Nevin and Baum, 1980; Myerson and Miezin, 1980). Let I equal the reciprocal of this sum: $I = 1/(\mu_1 + \mu_2)$. Next, note that if $p = \mu_2/(\mu_1 + \mu_2)$ then $1/\mu_1 = I/(1-p)$ and $1/\mu_2 = I/p$. That is, the average time at side 1 is $I/(1-p)$ and the average time at side 2 is I/p. Therefore changeover rate can be rewritten as

$$\frac{2}{E(t_1) + E(t_2)} = \frac{2}{\dfrac{I}{p} + \dfrac{I}{1-p}} = \frac{2p(1-p)}{I} \quad . \qquad (20B.1)$$

APPENDIX 20C

Assume an interdependent concurrent VT VT schedule with exponentially distributed intervals and a Poisson switching process. If the interval at the attended side elapses, the reinforcement is delivered immediately and the timer at the unattended side is not halted. In contrast, if the timer at the unattended side sets up, both timers are halted. The conditional probability of this occurring was calculated previously (see Eq. (20.1)). The result is $1/(1 + V_j \mu_i)$. Therefore the expected reinforcement rate at side 1 is

$$\frac{p}{V_1} + \frac{1-p}{V_1 + 1/\mu_2} - \frac{p}{V_1 (1 + V_2 \mu_1)} \quad . \qquad (20C.1)$$

For side 2 the expected rate is

$$\frac{1-p}{V_2} + \frac{p}{V_2 + 1/\mu_1} - \frac{1-p}{V_2 (1 + V_1 \mu_2)} \quad . \qquad (20C.2)$$

Note that reinforcement rate goes to zero if the subject does not switch (for example, $p = 0$ and $\mu_2 = 0$).

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